Nuclear Symmetry Energy from QCD sum rules

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We calculate the nucleon self-energies in an isospin asymmetric nuclear matter using QCD sum rules. Taking the difference of these for the neutron and proton enables us to express the potential part of the nuclear symmetry energy in terms of local operators. We find that the scalar (vector) self energy part gives negative (positive) contribution to the nuclear symmetry energy which are consistent with the results from relativistic mean field theories. Moreover, we find that an important contribution to the negative contribution of the scalar self energy comes from the twist-4 matrix elements, whose leading density dependence can be extracted from deep inelastic scattering experiments. This suggests that twist-4 contribution partly mimics the exchange of the δ meson and that it constitutes an essential part in the origin of the nuclear symmetry energy from QCD. Our result also extends an early success of QCD sum rule method in understanding the symmetric nuclear matter in terms of QCD variables to the asymmetric nuclear matter case.

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I. INTRODUCTION

With the recent constructions and plans for the next generation low energy rare isotope accelerators worldwide, there is a renewed interest in the nuclear symmetry energy, as the topic could be studied further with finer details in these experiments [1]. Understanding the details of the nuclear symmetry energy will provide valuable insights into exotic nuclear matter ranging from rare isotopes to neutron rich nuclear matter such as the neutron star [2, 3]. One of the main puzzle to be solved is the behavior of the nuclear symmetry energy at high density [4, 5].

From a phenomenological point of view, insights into nuclear symmetry energy can be obtained from understanding the nuclear binding expressed through the semi empirical mass formula [6]. There, the symmetry energy can be understood as originating from the energy difference between the proton and the neutron in an isospin asymmetric nuclear matter. Hence, within the approximation, the problem of calculating the nuclear symmetry energy can be reduced to obtaining the optical potential and thereby calculating the energy of the nucleon quasiparticle near the Fermi surfaces in an asymmetric nuclear matter.

In Dirac phenomenology of nucleon-nucleus scattering [7, 8], the optical potential of the nucleon is composed of a vector and scalar part, $U \simeq S + V\gamma^0$. It is well known that in order to fit the spin observables, one needs a strong scalar attraction (Re $S < 0$) and a strong vector repulsion (Re $V > 0$) both of several hundred MeV, but such that the combined sum gives only a small attraction of few tens of MeV, consistent with the traditional low energy nuclear physics. The strong scalar and vector potential comes about naturally also in relativistic mean field theories (RMFT), where meson exchange interaction between nucleons on the Fermi sea produce strong scalar and vector potentials for the nucleons.

But it was only after the works in QCD sum rules that the strong optical potentials were found to have a basis from QCD. The first pioneering work of applying QCD sum rule method to nucleons in medium were performed by Drukarev and Levin [9, 10]. Later, the relation became clearer through the work by Cohen, Furnstahl and Griegel [11], who used the energy dispersion relation to show that the showed the strong scalar-vector self energy appearing in the quasi-nucleon pole in the symmetric nuclear matter [11–14] can be traced back to the scalar-vector quark condensate in the nuclear medium and that the contribution from higher dimensional condensates do not change the general behavior.

Motivated by these results, and to express and elucidate the origin of nuclear symmetry energy directly from QCD, we have applied QCD sum rule to calculate the neutron and proton energy in the asymmetric nuclear matter. Identifying the difference with appropriate factors to the nuclear symmetry energy, we show that this energy can be expressed in terms of quark and gluon degree of freedom. Attempts to calculate the nucleon mass in an asymmetric matter using QCD sum rules were reported before [15–17]. But here, we will follow the formalism adopted in Ref. [11]. We have performed the operator product expansion (OPE) up to dimension 6 operators and have identified all the independent twist-4 operators, where for most of the operators, the leading density dependence can be extracted from the deep inelastic scattering data on higher twist effects. From the QCD sum rule analysis, we find that the scalar (vector) self energy part gives negative (positive) contribution to the nuclear symmetry energy which are consistent with the results from relativistic mean field theories. Moreover, we find that an important contribution to the negative contribution of the scalar self energy comes from the twist-4 matrix elements, whose higher density behavior will determine the still controversial property of the symmetry energy at these densities. Our result also extends an early success of QCD sum rule method in understanding the symmetric nuclear matter in terms of

QCD operators to the asymmetric nuclear matter case.

The paper is organized as follows. In Sec. II, we start with brief review and simple idea for the nuclear symmetry energy. QCD sum rule for nucleons in the asymmetric nuclear matter is presented in Sec. III. Finally, sum rule analysis and conclusion are given in Sec. IV.

II. A SIMPLIFIED DESCRIPTION FOR THE NUCLEAR SYMMETRY ENERGY

We start from a finite nuclei with A nucleons. In the Bethe-Weizsäker formula for the nuclear binding energy given as

$$\begin{aligned} m_{\rm tot} &= Nm_n + Zm_p - E_B/c^2, \\ E_B &= a_V A - a_S A^{\frac{2}{3}} - a_C(Z(Z-1))A^{-\frac{1}{3}} \\ &\quad - a_A I^2 A + \delta(A,Z), \end{aligned} \tag{1}$$

where $I = (N-Z)/A$, the fourth term accounts for the total shifted energy due to the neutron number excess. Taking the infinite nuclear matter limit of this formula, one notes that a_A reduces to the nuclear symmetry energy [6].

To derive the formula for a_A that can be generalized to the infinite nuclear matter, we start from a simple formula for the total energy:

$$\begin{aligned} E_{\rm tot} &= N\overline{E}_n + Z\overline{E}_p \\ &= \frac{1}{2}A(1+I)\overline{E}_n + \frac{1}{2}A(1-I)\overline{E}_p \\ &= \frac{1}{2}A(\overline{E}_n + \overline{E}_p) + \frac{1}{2}AI(\overline{E}_n - \overline{E}_p), \end{aligned} \tag{2}$$

where $\overline{E}_n$ ($\overline{E}_p$) is the average neutron (proton) quasi particle energy in the asymmetric nuclear matter. Now, the core of the model is what approximation goes into calculating the average energy.

The symmetry energy in an asymmetric nuclear matter is defined as

$$E_{\rm tot}(\rho, I) = E_0(\rho)A + E_{\rm sym}(\rho)I^2 A + O(I^4), \tag{3}$$

where ρ is the nuclear medium density and $I = (N-Z)/A \to (\rho_n - \rho_p)/(\rho_n + \rho_p)$ and the neutron and proton densities are $\rho_n = \frac{1}{2}\rho(1+I)$, $\rho_p = \frac{1}{2}\rho(1-I)$ respectively. Therefore, in Eq. (2), the symmetry energy will have contributions from the term proportional to I in $(\overline{E}_n - \overline{E}_p)$ and the term proportional to I^2 in $(\overline{E}_n + \overline{E}_p)$.

For a non interacting Fermi gas of nucleons with mass m_N, calculating the average nucleon energy will give $\overline{E} = \frac{3}{5}E_F$, where E_F is the nucleon Fermi energy. Following the procedure described above and extracting the term proportional to I^2 gives a nuclear symmetry energy of $\frac{1}{3}E_F$.

Going back to finite nuclei, assuming a 'Fermi well' with constant energy difference Δ between adjacent nucleon energy level, the symmetry energy can be obtained by from the second term Eq. (2). That is, $(\overline{E}_n - \overline{E}_p) = \frac{1}{4}IA\Delta$, hence,

$$a_A = \frac{1}{8}A\Delta = \frac{1}{4I}(E_n(A,I) - E_p(A,I)). \tag{4}$$

For infinite nuclear matter case, one can get $\overline{E}_N$ as:

$$\overline{E}_N = \frac{1}{\int d^3k_n d^3k_p} \int d^3k_n d^3k_p E_N(\rho_n, \rho_p). \tag{5}$$

And the nuclear symmetry energy can be obtained by collecting coefficients of I^2 in (2). This generalized $E^{\rm sym}(\rho)$ in Eq. (3) can be decomposed into kinetic like part and potential like part by mean field type quasi-particle approximation, in which one can treat both parts separately. The kinetic part of $E^{\rm sym}$ can be obtained according to Ref. [18];

$$E_K^{\rm sym} = \frac{1}{6}\frac{k_F^2}{\sqrt{k_F^2 + E_{q,V(I=0)}^2}}, \tag{6}$$

where k_F is the Fermi momentum and $E_{q,V(I=0)}$ is the potential part of the quasi-nucleon self energy in symmetric nuclear matter at saturation density.

A. Linear density approximation

In QCD sum rule calculations, we will be using the linear density approximation, because in medium condensates in QCD sum rule can be most reliably estimated to leading order in density. This means that for either the proton or the neutron, the mass will be given as follows:

$$\begin{aligned} E_V^n(\rho_n, \rho_p) &= m_0 + a\rho_p + b\rho_n \\ &= m_0 + \frac{1}{2}\rho(a+b) + \frac{1}{2}\rho I(b-a), \\ E_V^p(\rho_n, \rho_p) &= m_0 + \frac{1}{2}\rho(a+b) - \frac{1}{2}\rho I(b-a), \end{aligned} \tag{7}$$

where m_0 is the vacuum mass and a, b are the constants to be determined later. We then have,

$$\begin{aligned} \overline{E}_V^N &= \frac{1}{\int d^3k_n d^3k_p} \int d^3k_n d^3k_p E_V^N(\rho_n, \rho_p) \\ &= \left(m_0 + \frac{1}{2}a\rho_p + \frac{1}{2}b\rho_n\right) \\ &= \left(m_0 + \frac{1}{4}\rho(a+b) + \frac{1}{4}\rho I(b-a)\right). \end{aligned} \tag{8}$$

Combining Eq. (8) with Eq. (2), we obtain the symmetry energy. That is, $(\overline{E}_V^n - \overline{E}_V^p) = \frac{1}{2}(E_V^n(\rho_n, \rho_p) - E_V^p(\rho_n, \rho_p))$, hence,

$$E_V^{\rm sym} = \frac{1}{4I}(E_V^n(\rho_n, \rho_p) - E_V^p(\rho_n, \rho_p)), \tag{9}$$

is similar to the relation given in Eq. (4). Therefore, to this order, the symmetry energy comes only from the energy difference in the proton and neutron at the Fermi surface in an asymmetric nuclear matter as given in Eq. (9). However, for operators with higher dimension and higher density dependence, the factors appearing in Eq. (8) should be modified and symmetry energy will have contributions from both the sum and the difference of the nucleon energies.

The quantity of interest, namely, $(E_V^n(\rho_n, \rho_p) - E_V^p(\rho_n, \rho_p))$ can be obtained by looking at the pole of the nucleon propagator in nuclear medium:

$$G(q) = -i \int d^4x e^{iqx} \langle \Psi_0 | \mathrm{T}[\psi(x)\bar{\psi}(0)] | \Psi_0 \rangle, \tag{10}$$

where $|\Psi_0\rangle$ is the nuclear medium ground state, and $\psi(x)$ is a nucleon field. Relativistic mean field type of contribution will then appear in the self energies. The nucleon propagator can be decomposed as

$$G(q) = G_s(q^2, q \cdot u) + G_q(q^2, q \cdot u)\not{q} + G_u(q^2, q \cdot u)\not{u}, \tag{11}$$

where u^μ is the four-velocity of the nuclear medium ground state [12].

The nucleon self energy can be decomposed similarly as [11–14]:

$$\Sigma(q) = \tilde{\Sigma}_s(q^2, q \cdot u) + \tilde{\Sigma}_v^\mu(q)\gamma_\mu, \tag{12}$$

where

$$\tilde{\Sigma}_v^\mu(q) = \Sigma_u(q^2, q \cdot u)u^\mu + \Sigma_q(q^2, q \cdot u)q^\mu. \tag{13}$$

In the mean field approximation Σ_s and Σ_v are real and momentum independent, and Σ_q is negligible. Hence,

$$\Sigma_v \equiv \frac{\Sigma_u}{1 - \Sigma_q} \sim \Sigma_u, \quad M_N^* \equiv \frac{M_N + \tilde{\Sigma}_s}{1 - \Sigma_q} \sim M_N + \tilde{\Sigma}_s. \tag{14}$$

The phenomenological representation of the nucleon propagator can then be written as

$$G(q) = \frac{1}{\not{q} - M_n - \Sigma(q)} \to \lambda^2 \frac{\not{q} + M^* - \not{u}\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_0)}, \tag{15}$$

λ is unity in this discussion, but if one did not neglect Σ_q, effects of Σ_q should be accounted in λ^2 as the factor $(1 - \Sigma_q)^{-1}$. E_q and $\bar{E}_q$ are the positive and negative energy pole:

$$E_q = \Sigma_v + \sqrt{\vec{q}^2 + M_N^{*2}}, \tag{16}$$

$$\bar{E}_q = \Sigma_v - \sqrt{\vec{q}^2 + M_N^{*2}}. \tag{17}$$

With fixed $|\vec{q}|$, $G(q)$ only depends on q_0. One can extract self energy near $\sim E_q$ with analytic properties of the nucleon propagator.

III. QCD SUM RULE FOR NUCLEONS IN THE ASYMMETRIC NUCLEAR MEDIUM

A. Operator Product Expansion and Borel sum rule

To express the self energies in terms of QCD variables, we start with analyzing the correlation function via the operator product expansion (OPE). The Correlator is defined as

$$\Pi(q) \equiv i \int d^4x e^{iqx} \langle \Psi_0 | \mathrm{T}[\eta(x)\bar{\eta}(0)] | \Psi_0 \rangle, \tag{18}$$

where $\eta(x)$ is an interpolating field of nucleon and $|\Psi_0\rangle$ is the ground state of the asymmetric nuclear medium labeled with the rest frame medium density ρ, the medium four-velocity u^μ and the asymmetric factor I. This ground state is assumed to be invariant under parity and time reversal. We will be using the Ioffe nucleon interpolating field given as [11, 19],

$$\eta(x) = \epsilon_{abc}[u_a^T(x)C\gamma_\mu u_b(x)]\gamma_5\gamma^\mu d_c(x). \tag{19}$$

As in the case of the nucleon propagator, using Lorentz covariance, parity and time reversal, one can decompose the correlator into three invariants [12]:

$$\Pi(q) \equiv \Pi_s(q^2, q \cdot u) + \Pi_q(q^2, q \cdot u)\not{q} + \Pi_u(q^2, q \cdot u)\not{u}. \tag{20}$$

The three invariants are functions of q^2 and $q \cdot u$, while vacuum invariants depends only on q^2. For convenience, we set the rest frame for the nuclear medium, which means $u^\mu \to (1, \vec{0})$; also we fix $|\vec{q}|$. $\Pi_i(q^2, q \cdot u)$ then becomes a function of q_0 only, which means $\Pi_i(q^2, q \cdot u) \to \Pi_i(q_0, |\vec{q}| \to \text{fixed})$ $(i = \{s, q, u\})$.

As mentioned before, we will follow the formalism adopted in Ref. [11] and write the energy dispersion relation for these invariant functions at fixed three momentum $|\vec{q}|$:

$$\Pi_i(q_0, |\vec{q}|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{\Delta\Pi_i(\omega, |\vec{q}|)}{\omega - q_0} + \text{polynomials}, \tag{21}$$

$$\begin{aligned} \Delta\Pi_i(\omega, |\vec{q}|) &\equiv \lim_{\epsilon \to 0^+} [\Pi_i(\omega + i\epsilon, |\vec{q}|) - \Pi_i(\omega - i\epsilon, |\vec{q}|)] \\ &= 2\mathrm{Im}[\Pi_i(\omega, |\vec{q}|)]. \end{aligned} \tag{22}$$

The lowest energy contribution to the discontinuity will be saturated by a quasi-nucleon and quasi-hole contribution in the positive and negative energy domain respectively. Their contribution to the spectral density will be given as in Eq. (15), which will have the following con-

tribution to the invariant functions:

$$\Pi_s(q_0, |\vec{q}|) = -\lambda_N^{*2} \frac{M_N^*}{(q_0 - E_q)(q_0 - \bar{E}_q)} + \cdots, \quad (23)$$

$$\Pi_q(q_0, |\vec{q}|) = -\lambda_N^{*2} \frac{1}{(q_0 - E_q)(q_0 - \bar{E}_q)} + \cdots, \quad (24)$$

$$\Pi_u(q_0, |\vec{q}|) = +\lambda_N^{*2} \frac{\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_q)} + \cdots, \quad (25)$$

where λ_N^{*2} is the residue at the quasi-nucleon pole, which accounts for the coupling of the interpolating field to the quasi-nucleon excitation state, and the omitted parts are the contributions from the higher excitation states, which will be accounted for through the continuum contribution after the Borel transformation.

The even and odd parts of the invariant functions are respectively related to the following parts of the discontinuity:

$$\Pi_i^E(q_0^2, |\vec{q}|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{\omega \Delta\Pi_i(\omega, |\vec{q}|)}{\omega^2 - q_0^2} + \text{polynomials},$$
$$\Pi_i^O(q_0^2, |\vec{q}|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{\Delta\Pi_i(\omega, |\vec{q}|)}{\omega^2 - q_0^2} + \text{polynomials}, \quad (26)$$

where we have defined invariants as follow for convenience

$$\Pi_i(q_0, |\vec{q}|) = \Pi_i^E(q_0^2, |\vec{q}|) + q_0 \Pi_i^O(q_0^2, |\vec{q}|). \quad (27)$$

The OPE of the three invariants of both the even and odd parts can be expressed as

$$\Pi_i(q^2, q_0^2) = \sum_n C_n^i(q^2, q_0^2) \langle \hat{O}_n \rangle_{\rho,I}, \quad (28)$$

where $\langle \hat{O}_n \rangle_{\rho,I}$ is the ground state expectation value of the physical operator in the asymmetric nuclear medium; $\langle \Psi_0 | \hat{O}_n | \Psi_0 \rangle_{\rho,I}$. We will be adopting the OPE at $q^2 \to -\infty$ at finite $|\vec{q}| \to$ fixed; this is equivalent to the limit of $q_0^2 \to -\infty$ at finite $|\vec{q}| \to$ fixed. The Wilson coefficients $C_n^i(q^2, q_0)$ can thus be calculated in QCD at short time [11].

The OPE of the correlator for the proton interpolating field up to dimension 5 operators are given as follows:

$$\Pi_s^E(q_0^2, |\vec{q}|) = \frac{1}{4\pi^2} q^2 \ln(-q^2) \langle \bar{d}d \rangle_{\rho,I} + \frac{4}{3\pi^2} \frac{q_0^2}{q^2} \langle \bar{d}\{iD_0 iD_0\}d \rangle_{\rho,I}, \quad (29)$$

$$\Pi_s^O(q_0^2, |\vec{q}|) = -\frac{1}{2\pi^2} \ln(-q^2) \langle \bar{d} iD_0 d \rangle_{\rho,I}, \quad (30)$$

$$\begin{aligned}
\Pi_q^E(q_0^2, |\vec{q}|) = &-\frac{1}{64\pi^4} (q^2)^2 \ln(-q^2) \\
&+ \left(\frac{1}{9\pi^2} \ln(-q^2) - \frac{4}{9\pi^2} \frac{q_0^2}{q^2} \right) \langle \bar{d}\{\gamma_0 iD_0\}d \rangle_{\rho,I} + \left(\frac{4}{9\pi^2} \ln(-q^2) - \frac{4}{9\pi^2} \frac{q_0^2}{q^2} \right) \langle \bar{u}\{\gamma_0 iD_0\}u \rangle_{\rho,I} \\
&- \frac{1}{32\pi^2} \ln(-q^2) \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rho,I} - \frac{1}{144\pi^2} \left(\ln(-q^2) - \frac{4q_0^2}{q^2} \right) \left\langle \frac{\alpha_s}{\pi} [(u \cdot G)^2 + (u \cdot \tilde{G})^2] \right\rangle_{\rho,I},
\end{aligned} \quad (31)$$

$$\begin{aligned}
\Pi_q^O(q_0^2, |\vec{q}|) = &\frac{1}{6\pi^2} \ln(-q^2) \left[\langle u^\dagger u \rangle_{\rho,I} + \langle d^\dagger d \rangle_{\rho I} \right] - \frac{2}{3\pi^2} \frac{q_0^2}{(q^2)^2} \langle \bar{u}\{\gamma_0 iD_0 iD_0\}u \rangle_{\rho,I} \\
&- \frac{2}{3\pi^2} \frac{q_0^2}{(q^2)^2} \langle \bar{d}\{\gamma_0 iD_0 iD_0\}d \rangle_{\rho,I} - \frac{2}{3\pi^2} \frac{1}{q^2} \langle \bar{u}\{\gamma_0 iD_0 iD_0\}u \rangle_{\rho,I} + \frac{1}{18\pi^2} \frac{1}{q^2} \langle g_s u^\dagger \sigma \cdot \mathcal{G} u \rangle_{\rho,I},
\end{aligned} \quad (32)$$

$$\begin{aligned}
\Pi_u^E(q_0^2, |\vec{q}|) = &\frac{1}{12\pi^2} q^2 \ln(-q^2) \left[7\langle u^\dagger u \rangle_{\rho,I} + \langle d^\dagger d \rangle_{\rho,I} \right] + \frac{3}{\pi^2} \frac{q_0^2}{q^2} \langle \bar{u}\{\gamma_0 iD_0 iD_0\}u \rangle_{\rho,I} + \frac{1}{\pi^2} \frac{q_0^2}{q^2} \langle \bar{d}\{\gamma_0 iD_0 iD_0\}d \rangle_{\rho,I} \\
&- \frac{1}{6\pi^2} \ln(-q^2) \langle g_s u^\dagger \sigma \cdot \mathcal{G} u \rangle_{\rho,I} + \frac{1}{12\pi^2} \ln(-q^2) \langle g_s d^\dagger \sigma \cdot \mathcal{G} d \rangle_{\rho,I},
\end{aligned} \quad (33)$$

$$\begin{aligned}
\Pi_u^O(q_0^2, |\vec{q}|) = &-\frac{4}{9\pi^2} \ln(-q^2) \langle \bar{d}\{\gamma_0 iD_0\}d \rangle_{\rho,I} - \frac{16}{9\pi^2} \ln(-q^2) \langle \bar{u}\{\gamma_0 iD_0\}u \rangle_{\rho,I} \\
&+ \frac{1}{36\pi^2} \ln(-q^2) \left\langle \frac{\alpha_s}{\pi} [(u \cdot G)^2 + (u \cdot \tilde{G})^2] \right\rangle_{\rho,I}.
\end{aligned} \quad (34)$$

The quark part and their flavor structure of the above OPE can be obtained by suitable substitutions of the cor-

responding OPE for the Σ given in Ref. [20]; by changing $q \to u$, $s \to d$, and neglecting terms proportional to m_s. Moreover, when both u and d quarks are identified to the generic light flavor q, our OPE also reduces to that given in Ref. [14].

Next task is identifying the nucleon self energies in the asymmetric nuclear medium. So we have to concentrate on the quasi-nucleon pole and not on the quasi-hole nor on the continuum excitations. To this end, we apply the Borel transformation with appropriate weighting function to the dispersion relation [12]:

$$\mathcal{B}[\Pi_i(q_0, |\vec{q}|)] = \frac{1}{2\pi i}\int_{-\omega_0}^{\omega_0} d\omega \;\; W(\omega)\Delta\Pi_i(\omega, |\vec{q}|), \quad (35)$$

$$W(\omega) = (\omega - \bar{E}_q)e^{-\omega^2/M^2}, \quad (36)$$

where $\bar{E}_q$ is the quasi-hole pole which will be assigned as phenomenological input. The weighting function will de-emphasize the contribution from the quasi-hole, and the Borel transformation suppress the continuum contribution. Using Eq. (26), the OPE side of the sum rule can be obtained by taking the Borel transformation of $\Pi_i(q_0, |\vec{q}|) = \Pi_i^E(q_0^2, |\vec{q}|) - \bar{E}_q \Pi_i^O(q_0^2, |\vec{q}|)$. Here, we define differential operator $\mathcal{B}$ for the Borel transformation of the OPE side as

$$\begin{aligned}\mathcal{B}[f(q_0^2, |\vec{q}|)] &\equiv \lim_{\substack{-q_0^2, n\to\infty \\ -q_0^2/n = M^2}} \frac{(-q_0^2)^{n+1}}{n!}\left(\frac{\partial}{\partial q_0^2}\right)^n f(q_0^2, |\vec{q}|) \\ &\equiv \hat{f}(M^2, |\vec{q}|),\end{aligned} \quad (37)$$

where M is the Borel mass [21].

The Borel transformed invariants which contain continuum corrections are as follows:

$$\begin{aligned}\bar{\mathcal{B}}[\Pi_s(q_0^2, |\vec{q}|)] &= \lambda_N^{*2} M_p^* e^{-(E_q^2 - \vec{q}^2)/M^2} \\ &= -\frac{1}{4\pi^2}(M^2)^2 E_1 \langle \bar{d}d\rangle_{\rho,I} - \frac{4}{3\pi^2}\vec{q}^2\langle \bar{d}\{iD_0 iD_0\}d\rangle_{\rho,I} L^{-\frac{4}{9}} \\ &\quad + \bar{E}_q\Big[-\frac{1}{2\pi^2}M^2 E_0 \langle \bar{d} iD_0 d\rangle_{\rho,I} L^{-\frac{4}{9}}\Big],\end{aligned} \quad (38)$$

$$\begin{aligned}\bar{\mathcal{B}}[\Pi_q(q_0^2, |\vec{q}|)] &= \lambda_N^{*2} e^{-(E_q^2 - \vec{q}^2)/M^2} \\ &= \frac{1}{32\pi^4}(M^2)^3 E_2 L^{-\frac{4}{9}} - \left(\frac{1}{9\pi^2}M^2 E_0 - \frac{4}{9\pi^2}\vec{q}^2\right)\langle \bar{d}\{\gamma_0 iD_0\}d\rangle_{\rho,I} L^{-\frac{4}{9}} \\ &\quad - \left(\frac{4}{9\pi^2}M^2 E_0 - \frac{4}{9\pi^2}\vec{q}^2\right)\langle \bar{u}\{\gamma_0 iD_0\}u\rangle_{\rho,I} L^{-\frac{4}{9}} \\ &\quad + \frac{1}{32\pi^2}M^2\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle_{\rho,I} E_0 L^{-\frac{4}{9}} + \frac{1}{144\pi^2}\left(M^2 E_0 - 4\vec{q}^2\right)\left\langle \frac{\alpha_s}{\pi}[(u\cdot G)^2 + (u\cdot\tilde{G})^2]\right\rangle_{\rho,I} L^{-\frac{4}{9}} \\ &\quad + \bar{E}_q\bigg[\frac{1}{6\pi^2}M^2 E_0 L^{-\frac{4}{9}}\left[\langle u^\dagger u\rangle_{\rho,I} + \langle d^\dagger d\rangle_{\rho,I}\right] - \frac{2}{3\pi^2}\left(1 - \frac{\vec{q}^2}{M^2}\right)\langle \bar{u}\{\gamma_0 iD_0 iD_0\}u\rangle_{\rho,I} L^{-\frac{4}{9}} \\ &\qquad - \frac{2}{3\pi^2}\left(1 - \frac{\vec{q}^2}{M^2}\right)\langle \bar{d}\{\gamma_0 iD_0 iD_0\}d\rangle_{\rho,I} L^{-\frac{4}{9}} \\ &\qquad - \frac{2}{3\pi^2}\langle \bar{u}\{\gamma_0 iD_0 iD_0\}u\rangle_{\rho,I} L^{-\frac{4}{9}} + \frac{1}{18\pi^2}\langle g_s u^\dagger \sigma\cdot\mathcal{G}u\rangle_{\rho,I} L^{-\frac{4}{9}}\bigg],\end{aligned} \quad (39)$$

$$\begin{aligned}\bar{\mathcal{B}}[\Pi_u(q_0^2, |\vec{q}|)] &= \lambda_N^{*2}\Sigma_v^p e^{-(E_q^2 - \vec{q}^2)/M^2} \\ &= \frac{1}{12\pi^2(M^2)^2}\left[7\langle u^\dagger u\rangle_{\rho,I} + \langle d^\dagger d\rangle_{\rho,I}\right] E_1 L^{-\frac{4}{9}} + \frac{3}{\pi^2}\vec{q}^2\langle \bar{u}\{\gamma_0 iD_0 iD_0\}u\rangle_{\rho,I} L^{-\frac{4}{9}} \\ &\quad + \frac{1}{\pi^2}\vec{q}^2\langle \bar{d}\{\gamma_0 iD_0 iD_0\}d\rangle_{\rho,I} L^{-\frac{4}{9}} - \frac{1}{6\pi^2}M^2\langle g_s u^\dagger\sigma\cdot\mathcal{G}u\rangle_{\rho,I} E_0 L^{-\frac{4}{9}} \\ &\quad + \frac{1}{12\pi^2}M^2\langle g_s d^\dagger\sigma\cdot\mathcal{G}d\rangle_{\rho,I} E_0 L^{-\frac{4}{9}} \\ &\quad + \bar{E}_q\bigg[-\frac{4}{9\pi^2}M^2\langle \bar{d}\{\gamma_0 iD_0\}d\rangle_{\rho,I} E_0 L^{-\frac{4}{9}} - \frac{16}{9\pi^2}M^2\langle \bar{u}\{\gamma_0 iD_0\}u\rangle_{\rho,I} E_0 L^{-\frac{4}{9}} \\ &\qquad - \frac{1}{36\pi^2}M^2\left\langle \frac{\alpha_s}{\pi}[(u\cdot G)^2 + (u\cdot\tilde{G})^2]\right\rangle_{\rho,I} E_0 L^{-\frac{4}{9}}\bigg].\end{aligned} \quad (40)$$

Here, we include the contributions from the anomalous-dimensions as

$$L^{-2\Gamma_\eta+\Gamma_{O_n}} \equiv \left[\frac{\ln(M/\Lambda_{QCD})}{\ln(\mu/\Lambda_{QCD})}\right]^{-2\Gamma_\eta+\Gamma_{O_n}}, \tag{41}$$

where Γ_η (Γ_{O_n}) is the anomalous dimension of the interpolating field η ($\hat{O}_n$), μ is the normalization point of the OPE, and Λ_{QCD} is the QCD scale [12, 14].

Also, the continuum corrections are taken into account through the factors

$$E_0 \equiv 1 - e^{s_0^*/M^2}, \tag{42}$$
$$E_1 \equiv 1 - e^{s_0^*/M^2}\left(s_0^*/M^2+1\right), \tag{43}$$
$$E_2 \equiv 1 - e^{s_0^*/M^2}\left(s_0^{*2}/2M^4 + s_0^*/M^2+1\right), \tag{44}$$

where $s_0^* \equiv \omega_0^2 - \vec{q}^2$, and ω_0 is the energy at the continuum threshold. As in many previous studies, we choose $\omega_0 = 1.5$ GeV.

B. Condensates in the asymmetric nuclear medium

To estimate the matrix elements, we will use the linear density approximation in the asymmetric nuclear matter:

$$\begin{aligned}\langle\hat{O}\rangle_{\rho,I} =&\ \langle\hat{O}\rangle_{\rm vac} + \langle n|\hat{O}|n\rangle\rho_n + \langle p|\hat{O}|p\rangle\rho_p \\ =&\ \langle\hat{O}\rangle_{\rm vac} + \frac{1}{2}\Big(\langle n|\hat{O}|n\rangle + \langle p|\hat{O}|p\rangle\Big)\rho + \frac{1}{2}\Big(\langle n|\hat{O}|n\rangle - \langle p|\hat{O}|p\rangle\Big)I\rho.\end{aligned} \tag{45}$$

The quark flavor of condensate becomes important in the asymmetric nuclear medium. Consider an operator $\hat{O}_{u,d}$ composed of either u or d quark respectively. Making use of the isospin symmetry relation,

$$\langle n|\hat{O}_{u,d}|n\rangle = \langle p|\hat{O}_{d,u}|p\rangle, \tag{46}$$

we can convert the neutron expectation value to the proton expectation value, thereby rewriting Eq. (45) for the two quark operators as follows:

$$\langle\hat{O}_{u,d}\rangle_{\rho,I} = \langle\hat{O}_{u,d}\rangle_{\rm vac} + (\langle p|\hat{O}_0|p\rangle \mp \langle p|\hat{O}_1|p\rangle I)\rho. \tag{47}$$

Here, '$-$' and '$+$' are for 'u' and 'd' quark flavor respectively, and the isospin operators are defined as

$$\hat{O}_0 \equiv \frac{1}{2}(\hat{O}_u + \hat{O}_d), \quad \hat{O}_1 \equiv \frac{1}{2}(\hat{O}_u - \hat{O}_d). \tag{48}$$

Hence, we will convert all the expectation values in terms of the proton counterparts and denote them as $\langle p|\hat{O}|p\rangle \to \langle\hat{O}\rangle_p$, throughout this paper. The next task is to find $\langle\hat{O}_0\rangle_p$ and $\langle\hat{O}_1\rangle_p$ for all operators appearing in our OPE.

1. $\langle\bar{q}D_{\mu_1}\cdots D_{\mu_n}q\rangle$ type of condensates

Let us start by estimating the lowest dimensional operators $\langle[\bar{q}q]_0\rangle_p$ and $\langle[\bar{q}q]_1\rangle_p$. To find $\langle[\bar{q}q]_1\rangle_p$, we will use an estimate based on using the QCD energy momentum tensor in the baryon octet mass relation to leading order in the quark mass[22]; Eq. (A3) in Appendix A. Using Eq. (A4), one finds

$$\langle[\bar{q}q]_1\rangle_p = \frac{1}{2}\left(\langle p|\bar{u}u|p\rangle - \langle p|\bar{d}d|p\rangle\right) = \frac{1}{2}\left(\frac{(m_{\Xi^0}+m_{\Xi^-}) - (m_{\Sigma^+}+m_{\Sigma^-})}{2m_s - 2m_q}\right). \tag{49}$$

We will use the baryon masses as given in the Particle Data Group [23]: $m_{\Xi^0} = 1315$ MeV, $m_{\Xi^-} = 1321$ MeV, $m_{\Sigma^+} = 1190$ MeV, $m_{\Sigma^-} = 1197$ MeV. Using $m_s = 150$ MeV and $m_q \equiv \frac{1}{2}(m_u + m_d) = 5$ MeV, Eq. (49) becomes

$$\langle[\bar{q}q]_1\rangle_p = \frac{1}{2}\left(\frac{249\text{ MeV}}{300\text{ MeV} - 2m_q}\right) \sim 0.43. \tag{50}$$

For $\langle[\bar{q}q]_0\rangle_p$, we make use of the nucleon $\sigma_N = 45$ MeV term:

$$\langle[\bar{q}q]_0\rangle_p = \frac{1}{2}\left(\langle p|\bar{u}u|p\rangle + \langle p|\bar{d}d|p\rangle\right) = \frac{\sigma_N}{2m_q} \sim 4.5. \tag{51}$$

For convenience, one can introduce the parameter $\mathcal{R}_\pm(m_q)$ defined as

$$\langle p|\bar{u}u|p\rangle \pm \langle p|\bar{d}d|p\rangle = \mathcal{R}_\pm(m_q)\cdot\langle p|\bar{u}u|p\rangle, \tag{52}$$

which leads to

$$\langle[\bar{q}q]_1\rangle_p = \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)}\cdot\langle[\bar{q}q]_0\rangle_p. \tag{53}$$

Using the previously selected values with the explicit quark mass dependence, we have

$$\mathcal{R}_\pm(m_q) \equiv \left(1 \pm \left(\frac{\sigma_N}{m_q} - \frac{249\text{ MeV}}{300\text{ MeV} - 2m_q}\right) \Big/ \left(\frac{\sigma_N}{m_q} + \frac{249\text{ MeV}}{300\text{ MeV} - 2m_q}\right)\right), \tag{54}$$

so that $\mathcal{R}_\pm(m_q = 5\text{ MeV})) = 1 \pm 0.68$.

Using this parametrization, we can express the u quark or d quark condensates as follows,

$$\langle[\bar{q}q]_{u,d}\rangle_{\rho,I} = \langle[\bar{q}q]_{u,d}\rangle_{\text{vac}} + \left(1 \mp \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)}I\right)\cdot\langle[\bar{q}q]_0\rangle_p\ \rho, \tag{55}$$

where $[\bar{q}q]_u = \bar{u}u$ and $[\bar{q}q]_d = \bar{d}d$. For $\langle\bar{q}q\rangle_{\text{vac}}$, we use the Gellmann-Oakes-Renner relation:

$$2m_q\langle\bar{q}q\rangle_{\text{vac}} = -m_\pi^2 f_\pi^2, \tag{56}$$

where $m_\pi = 138$ MeV and $f_\pi = 98$ MeV [14]. For $m_q = 5$ MeV, we have $\langle\bar{q}q\rangle_{\text{vac}} = -(263\text{ MeV})^3$

Likewise, we will further assume that the ratios between the isospin singlet and triplet operators remain the same for all two quark operator expectation values with any number of covariant derivatives inserted:

$$\langle[\bar{q}D_{\mu_1}\cdots D_{\mu_n}q]_1\rangle_p = \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)}\cdot\langle[\bar{q}D_{\mu_1}\cdots D_{\mu_n}q]_0\rangle_p. \tag{57}$$

With this assumption, $\langle\bar{q}D_{\mu_1}\cdots D_{\mu_n}q\rangle_{\rho,I}$ can be written as

$$\langle[\bar{q}D_{\mu_1}\cdots D_{\mu_n}q]_{u,d}\rangle_{\rho,I} = \langle[\bar{q}D_{\mu_1}\cdots D_{\mu_n}q]_{u,d}\rangle_{\text{vac}} + \left(1 \mp \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)}I\right)\cdot\langle[\bar{q}D_{\mu_1}\cdots D_{\mu_n}q]_0\rangle_p\ \rho. \tag{58}$$

The symmetric and traceless part of the above type of expectation values, constitute the moments of the twist-3 $e_n(x,\mu^2)$ structure function defined as follows[36]:

$$\langle[\bar{q}\{D_{\mu_1}\cdots D_{\mu_n}\}q]_0\rangle_p \equiv (-i)^n e_n(\mu^2)\{p_{\mu_1}\cdots p_{\mu_n}\}, \tag{59}$$

$$e_n(\mu^2) \equiv \int dx\ x^n e_n(x,\mu^2), \tag{60}$$

where $\{\mu_1\cdots\mu_n\}$ means symmetric and traceless indices. Then the two quark twist-3 condensates in our sum rule can be written as follows:

$$\langle[\bar{q}iD_{\mu'}q]_{u,d}\rangle_{\rho,I} = \langle[\bar{q}iD_0q]_{u,d}\rangle_{\rho,I}\cdot u'_\mu =\ m_q\langle[q^\dagger q]_{u,d}\rangle_{\rho,I} = 0, \tag{61}$$

$$\langle[\bar{q}\{iD_{\mu'}iD_{\nu'}\}q]_{u,d}\rangle_{\rho,I} = \frac{4}{3}\langle[\bar{q}\{iD_0iD_0\}q]_{u,d}\rangle_{\rho,I}\cdot\left(u'_\mu u'_\nu - \frac{1}{4}g_{\mu\nu}\right), \tag{62}$$

where the in-medium rest frame $u'_\mu \equiv (1,\vec{0})$ has been taken and the matrix element is estimated as

$$\langle[\bar{q}\{iD_{\mu'}iD_{\nu'}\}q]_0\rangle_p =\ M_N^2 e_2(\mu^2)\left(u'_\mu u'_\nu - \frac{1}{4}g_{\mu\nu}\right), \tag{63}$$

where one can identify that $M_N^2 e_2(\mu^2) = \frac{4}{3}\langle[\bar{q}\{iD_0 iD_0\}q]_0\rangle_p$ and $\langle[\bar{q}\{iD_0 iD_0\}q]_{u,d}\rangle_{\rho,I}$ can be written as

$$\langle[\bar{q}\{iD_0 iD_0\}q]_{u,d}\rangle_{\rho,I} \simeq \left(1 \mp \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)} I\right) \cdot M_N^2 e_2(\mu^2)\ \rho. \tag{64}$$

Since there are no measurements on the twist-3 structure function, we will take the estimate for $M_N^2 e_2(\mu^2) \sim 0.3$ GeV^2 given in [14, 26].

When spin indices are contracted, the operator becomes

$$\langle[\bar{q}D^2 q]_{u,d}\rangle_{\rho,I} = \frac{1}{2}\langle[g_s\bar{q}\sigma\cdot\mathcal{G}q]_{u,d}\rangle_{\rho,I} = \frac{1}{2}\left(1 \mp \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)} I\right) \cdot \langle[g_s\bar{q}\sigma\cdot\mathcal{G}q]_0\rangle_p\ \rho, \tag{65}$$

where $\langle[g_s\bar{q}\sigma\cdot\mathcal{G}q]_0\rangle_p$ is chosen to be 3 GeV^2 as in Ref. [14, 26].

2. $\langle\bar{q}\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_n}q\rangle$ type of condensates

The simplest condensate of this type is

$$\langle\bar{q}\gamma_\lambda q\rangle_{\rho,I} = \langle\bar{q}\not{u}' q\rangle_{\rho,I} u'_\lambda \rightarrow \langle q^\dagger q\rangle_{\rho,I} u'_\lambda. \tag{66}$$

For this, the ratio $\langle u^\dagger u\rangle_p/\langle d^\dagger d\rangle_p = 2$, and the iso-spin relation for $\langle q^\dagger q\rangle_{\rho,I}$ can be written as

$$\langle[q^\dagger q]_1\rangle_p = \frac{1}{3}\langle[q^\dagger q]_0\rangle_p, \tag{67}$$

which leads to the following matrix elements appearing in the sum rule:

$$\langle[q^\dagger q]_{u,d}\rangle_{\rho,I} = \left(1 \mp \frac{1}{3}I\right) \cdot \langle[q^\dagger q]_0\rangle_p\ \rho = \left(\frac{3}{2} \mp \frac{1}{2}I\right)\rho. \tag{68}$$

When covariant derivatives are included, one can estimate the two quark twist-2 condensates from the corresponding parton distribution function:

$$\langle\bar{q}\{\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_n}\}q\rangle_p \equiv \frac{(-i)^{n-1}}{2M_N} A_n^q(\mu^2)\{p_{\mu_1}\cdots p_{\mu_n}\}, \tag{69}$$

where $A_n^q(\mu^2) = (A_n^u(\mu^2) + A_n^d(\mu^2))/2$ is the reduced matrix element [37, 38]:

$$A_n^q(\mu^2) = 2\int_0^1 dx\ x^{n-1}[q(x,\mu^2) + (-1)^n\bar{q}(x,\mu^2)], \tag{70}$$

where $q(x,\mu^2)$ and $\bar{q}(x,\mu^2)$ are the distribution functions for quarks and antiquarks in the proton respectively. μ^2 is the renormalization scale. For the distribution functions, we used the leading order (LO) parametrization given in Ref. [39].

Specifically, the spin-2 part can be written as

$$\langle[\bar{q}\{\gamma_\mu iD_\nu\}q]_{u,d}\rangle_{\rho,I} \rightarrow \langle[\bar{q}\{\gamma_{\mu'} iD_{\nu'}\}q]_{u,d}\rangle_{\rho,I} = \frac{4}{3}\langle[\bar{q}\{\gamma_0 iD_0\}q]_{u,d}\rangle_{\rho,I} \cdot \left(u'_\mu u'_\nu - \frac{1}{4}g_{\mu\nu}\right), \tag{71}$$

where the in-medium rest frame has been taken. The matrix elements for each flavor in $\langle[\bar{q}\{\gamma_0 iD_0\}q]_{u,d}\rangle_p$ can be identified as

$$\langle\bar{u}\{\gamma_{\mu'} iD_{\nu'}\}u\rangle_p = \frac{1}{2}M_N A_2^u(\mu^2) \cdot \left(u'_\mu u'_\nu - \frac{1}{4}g_{\mu\nu}\right), \tag{72}$$

$$\langle\bar{d}\{\gamma_{\mu'} iD_{\nu'}\}d\rangle_p = \frac{1}{2}M_N A_2^d(\mu^2) \cdot \left(u'_\mu u'_\nu - \frac{1}{4}g_{\mu\nu}\right), \tag{73}$$

where $A_2^u(\mu^2) \simeq 0.74$ and $A_2^d(\mu^2) \simeq 0.36$ at $\mu^2 = 0.25$ GeV^2 (LO) [39].

One can introduce a ratio factor for $\langle \hat{O}_1 \rangle_p$ as

$$\langle [\bar{q}\{\gamma_{\mu'} iD_{\nu'}\}q]_1 \rangle_p = \mathcal{R}_{A_2}(\mu^2) \cdot \langle [\bar{q}\{\gamma_{\mu'} iD_{\nu'}\}q]_0 \rangle_p, \tag{74}$$

where $\mathcal{R}_{A_2}(\mu^2) = (A_2^u - A_2^d)/(A_2^u + A_2^d) \simeq 0.35$ so that $\langle [\bar{q}\{\gamma_0 iD_0\}q]_{u,d} \rangle_{\rho,I}$ can be written as

$$\begin{aligned} \langle [\bar{q}\{\gamma_0 iD_0\}q]_{u,d} \rangle_{\rho,I} &= \left(1 \mp \mathcal{R}_{A_2}(\mu^2) I\right) \cdot \langle [\bar{q}\{\gamma_0 iD_0\}q]_0 \rangle_p \ \rho \\ &= \left(1 \mp \mathcal{R}_{A_2}(\mu^2) I\right) \cdot \frac{1}{2} M_N A_2^q(\mu^2) \rho. \end{aligned} \tag{75}$$

The spin-3 part can be written as

$$\langle [\bar{q}\{\gamma_{\lambda'} iD_{\mu'} iD_{\nu'}\}q]_{u,d} \rangle_{\rho,I} = 2 \langle [\bar{q}\{\gamma_0 iD_0 iD_0\}q]_{u,d} \rangle_{\rho,I} \cdot \left(u'_\lambda u'_\mu u'_\nu - \frac{1}{6}(u'_\lambda g_{\mu\nu} + u'_\mu g_{\lambda\nu} + u'_\nu g_{\lambda\mu}) \right), \tag{76}$$

where the matrix elements for each flavor in $\langle [\bar{q}\{\gamma_0 iD_0 iD_0\}q]_{u,d} \rangle_p$ can be identified with

$$\langle \bar{u}\{\gamma_{\lambda'} iD_{\mu'} iD_{\nu'}\}u \rangle_p = \frac{1}{2} M_N A_3^u(\mu^2) \cdot \left(u'_\lambda u'_\mu u'_\nu - \frac{1}{6}(u'_\lambda g_{\mu\nu} + u'_\mu g_{\lambda\nu} + u'_\nu g_{\lambda\mu}) \right), \tag{77}$$

$$\langle \bar{d}\{\gamma_{\lambda'} iD_{\mu'} iD_{\nu'}\}d \rangle_p = \frac{1}{2} M_N A_3^d(\mu^2) \cdot \left(u'_\lambda u'_\mu u'_\nu - \frac{1}{6}(u'_\lambda g_{\mu\nu} + u'_\mu g_{\lambda\nu} + u'_\nu g_{\lambda\mu}) \right), \tag{78}$$

where $A_3^u(\mu^2) \simeq 0.22$ and $A_3^d(\mu^2) \simeq 0.07$ at $\mu^2 = 0.25$ GeV^2 (LO) [39]. Similar to the spin-2 condensate case, one can write $\langle \hat{O}_1 \rangle_p$ for spin-3 condensate as

$$\langle [\bar{q}\{\gamma_{\lambda'} iD_{\mu'} iD_{\nu'}\}q]_1 \rangle_p = \mathcal{R}_{A_3}(\mu^2) \cdot \langle [\bar{q}\{\gamma_{\lambda'} iD_{\mu'} iD_{\nu'}\}q]_0 \rangle_p, \tag{79}$$

where $\mathcal{R}_{A_3}(\mu^2) = (A_3^u - A_3^d)/(A_3^u + A_3^d) \simeq 0.51$ and $\langle [\bar{q}\{\gamma_0 iD_0 iD_0\}q]_{u,d} \rangle_{\rho,I}$ can be written as

$$\begin{aligned} \langle [\bar{q}\{\gamma_0 iD_0 iD_0\}q]_{u,d} \rangle_{\rho,I} &= \left(1 \mp \mathcal{R}_{A_3}(\mu^2) I\right) \cdot \langle [\bar{q}\{\gamma_0 iD_0 iD_0\}q]_0 \rangle_p \ \rho \\ &= \left(1 \mp \mathcal{R}_{A_3}(\mu^2) I\right) \cdot \frac{1}{2} M_N A_3^q(\mu^2) \rho. \end{aligned} \tag{80}$$

Operators with contracted spin indices are

$$\langle [\bar{q} \not{D} q]_{u,d} \rangle_{\rho,I} = 0, \tag{81}$$

$$\langle [q^\dagger D^2 q]_{u,d} \rangle_{\rho,I} = \frac{1}{2} \langle [g_s q^\dagger \sigma \cdot \mathcal{G} q]_{u,d} \rangle_{\rho,I} \simeq \frac{1}{2} (1 \mp \mathcal{R}_{A_3} I) \cdot \langle [g_s q^\dagger \sigma \cdot \mathcal{G} q]_0 \rangle_p \ \rho, \tag{82}$$

where $\langle [g_s q^\dagger \sigma \cdot \mathcal{G} q]_0 \rangle_p$ is chosen to be -0.33 GeV^2 [14, 26].

3. *Gluon condensates*

As for the gluon operators, because they do not carry quark flavors, the expectation values do not depend on I. These operators can be written as [13, 14]

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rho,I} = \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\text{vac}} - 2 \left\langle \frac{\alpha_s}{\pi} (\vec{E}^2 - \vec{B}^2) \right\rangle_p \rho, \tag{83}$$

$$\left\langle \frac{\alpha_s}{\pi} [(u \cdot G)^2 + (u \cdot \tilde{G})^2] \right\rangle_{\rho,I} = - \left\langle \frac{\alpha_s}{\pi} (\vec{E}^2 + \vec{B}^2) \right\rangle_p \rho, \tag{84}$$

where $\vec{E}$ and $\vec{B}$ are the color electric and color magnetic fields. For the expectation values of the gluon operators we take; $\langle (\alpha_s/\pi) G^2 \rangle_{\text{vac}} = (0.33 \text{ GeV})^4$ [27], $\langle (\alpha_s/\pi)(\vec{E}^2 - \vec{B}^2) \rangle_p = 0.325 \pm 0.075$ GeV and $\langle (\alpha_s/\pi)(\vec{E}^2 + \vec{B}^2) \rangle_p = 0.10 \pm 0.01$ GeV [13].

C. Dimension 6 four-quark operators

In many previous QCD sum rule studies, dimension 6 four-quark condensates are assumed to have the factorized form as

$$\langle u^a_\alpha \bar{u}^b_\beta u^c_\gamma \bar{u}^d_\delta\rangle_{\rho,I} \simeq \langle u^a_\alpha \bar{u}^b_\beta\rangle_{\rho,I}\langle u^c_\gamma \bar{u}^d_\delta\rangle_{\rho,I} - \langle u^a_\alpha \bar{u}^d_\delta\rangle_{\rho,I}\langle u^c_\gamma \bar{u}^b_\beta\rangle_{\rho,I}, \tag{85}$$

$$\langle u^a_\alpha \bar{u}^b_\beta d^c_\gamma \bar{d}^d_\delta\rangle_{\rho,I} \simeq \langle u^a_\alpha \bar{u}^b_\beta\rangle_{\rho,I}\langle d^c_\gamma \bar{d}^d_\delta\rangle_{\rho,I}. \tag{86}$$

While large N_c arguments can be made to justify factorization in the vacuum, no such argument exists in the medium. Using the following steps, one can classify the four-quark condensates in terms of the independent operators and of different twist.

1. Twist-4 operators with pure quark flavor

The first type of four-quark operator appearing in the OPE of the nucleon sum rule, involves quark operators with the same flavor, and is of the color anti-triplet diquark times triplet anti diquark form. Using the following Fierz transformation, one can identify the independent four-quark operators in terms of products of quark-antiquark pairs.

$$\begin{aligned}
\epsilon_{abc}\epsilon_{a'b'c}(u^T_a C\gamma_\mu u_b)(\bar{u}^T_{b'}\gamma_\nu C\bar{u}_{a'}) &= \epsilon_{abc}\epsilon_{a'b'c}\,\frac{1}{16}\,(\bar{u}_{a'}\Gamma^o u_a)(\bar{u}_{b'}\Gamma^k u_b)\cdot \mathrm{Tr}\left[\gamma_\mu\Gamma_k\gamma_\nu C\Gamma^T_o C\right]\\
&= \epsilon_{abc}\epsilon_{a'b'c}\,\frac{1}{16}\,\Big\{(\bar{u}_{a'}u_a)(\bar{u}_{b'}u_b)\cdot(-4g_{\mu\nu})\\
&\quad+(\bar{u}_{a'}\gamma_5 u_a)(\bar{u}_{b'}\gamma_5 u_b)\cdot(4g_{\mu\nu})\\
&\quad+(\bar{u}_{a'}\gamma^\alpha u_a)(\bar{u}_{b'}\gamma^\beta u_b)\cdot(4S_{\mu\beta\nu\alpha})\\
&\quad-(\bar{u}_{a'}\gamma^\alpha\gamma_5 u_a)(\bar{u}_{b'}\gamma^\beta\gamma_5 u_b)\cdot(4S_{\mu\beta\nu\alpha})\\
&\quad+(\bar{u}_{a'}\sigma^{\alpha\bar{\alpha}} u_a)(\bar{u}_{b'}\sigma^{\beta\bar{\beta}} u_b)\cdot\frac{1}{4}\mathrm{Tr}\left[\gamma_\mu\sigma_{\beta\bar{\beta}}\gamma_\nu\sigma_{\alpha\bar{\alpha}}\right]\\
&\quad+(\bar{u}_{a'}\gamma^\alpha u_a)(\bar{u}_{b'}\gamma^\beta\gamma_5 u_b)\cdot(8i\epsilon_{\mu\alpha\nu\alpha})\\
&\quad-(\bar{u}_{a'}u_a)(\bar{u}_{b'}\sigma^{\alpha\bar{\alpha}} u_b)\cdot(8ig_{\alpha\mu}g_{\bar{\alpha}\nu})\\
&\quad-(\bar{u}_{a'}\gamma_5 u_a)(\bar{u}_{b'}\sigma^{\alpha\bar{\alpha}} u_b)\cdot(4\epsilon_{\mu\nu\alpha\bar{\alpha}})\Big\},
\end{aligned} \tag{87}$$

where $\Gamma = \{I, \gamma_\alpha, i\gamma_\alpha\gamma_5, \sigma_{\alpha\beta}, \gamma_5\}$ and $S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\alpha\nu} - g_{\mu\nu}g_{\alpha\beta}$.

When quarks of the same flavor combine into a diquark, certain combinations are not allowed due to Fermi statistics. From these conditions, one can extract constraints among four-quark operators that can be used to identify independent operators. Among several conditions, the most suitable constraint for our OPE can be obtained from the zero identity used in Ref. [35]. With the constraint Eq. (B2) in Appendix B, Eq. (87) can be simplified as

$$\begin{aligned}
\epsilon_{abc}\epsilon_{a'b'c}(u^T_a C\gamma_\mu u_b)(\bar{u}^T_{b'}\gamma_\nu C\bar{u}_{a'}) = \ \epsilon_{abc}\epsilon_{a'b'c}\,\frac{1}{16}\,\Big\{&\left[(\bar{u}_{a'}\gamma^\alpha u_a)(\bar{u}_{b'}\gamma^\beta u_b) - (\bar{u}_{a'}\gamma^\alpha\gamma_5 u_a)(\bar{u}_{b'}\gamma^\beta\gamma_5 u_b)\right]\cdot(8S_{\mu\alpha\nu\alpha})\\
&+(\bar{u}_{a'}\gamma^\alpha u_a)(\bar{u}_{b'}\gamma^\beta\gamma_5 u_b)\cdot(16i\epsilon_{\mu\beta\nu\alpha})\Big\}.
\end{aligned} \tag{88}$$

The last term in Eq. (88) will be dropped as one should take expectation value with respect to parity-even nuclear medium ground state. Then, only two types of four-quark operators remain. Each type can be written as

$$\begin{aligned}
\epsilon_{abc}\epsilon_{a'b'c}(\bar{q}_{a'}\Gamma^\alpha q_a)(\bar{q}_{b'}\Gamma^\beta q_b) = \epsilon_{abc}\epsilon_{a'b'c}\Big\{&\frac{1}{9}\delta_{a'a}\delta_{b'b}(\bar{q}\Gamma^\alpha q)(\bar{q}\Gamma^\beta q) + \frac{2}{3}t^A_{aa'}\delta_{b'b}(\bar{q}\Gamma^\alpha t^A q)(\bar{q}\Gamma^\beta q)\\
&+\frac{2}{3}\delta_{a'a}t^B_{bb'}(\bar{q}\Gamma^\alpha q)(\bar{q}\Gamma^\beta t^B q) + 4t^A_{aa'}t^B_{bb'}(\bar{q}\Gamma^\alpha t^A q)(\bar{q}\Gamma^\beta t^B q)\Big\},
\end{aligned} \tag{89}$$

where $\Gamma^\alpha = \{\gamma^\alpha, i\gamma^\alpha\gamma_5\}$ and t^A is the generator of SU(3) normalized as $\mathrm{Tr}[t^A t^B] = \frac{1}{2}\delta^{AB}$. Combined with the product of epsilon tensors $\epsilon_{abc}\epsilon_{a'b'c} = \delta_{bb'}\delta_{aa'} - \delta_{ba'}\delta_{ab'}$, one finds that the second and third term in the R.H.S. of Eq. (89)

vanish. In the last term, the product of the generator of SU(3) can be simplified using the following identity:

$$t^A_{a'a}t^B_{b'b} = \frac{1}{8}\delta^{AB}t^C_{a'a}t^C_{b'b} + \left[t^A_{a'a}t^B_{b'b} - \frac{1}{8}\delta^{AB}t^C_{a'a}t^C_{b'b}\right], \tag{90}$$

where only the first term in the R.H.S. of Eq. (90) survives after multiplying it with the epsilon tensors $\epsilon_{abc}\epsilon_{a'b'c}$. Then Eq. (89) can be simplified as follows

$$\begin{aligned}\epsilon_{abc}\epsilon_{a'b'c}(\bar{q}_{a'}\Gamma^\alpha q_a)(\bar{q}_{b'}\Gamma^\beta q_b) &= \epsilon_{abc}\epsilon_{a'b'c}\left\{\frac{1}{9}\delta_{a'a}\delta_{b'b}(\bar{q}\Gamma^\alpha q)(\bar{q}\Gamma^\beta q) + \frac{1}{2}t^A_{aa'}t^A_{bb'}(\bar{q}\Gamma^\alpha t^B q)(\bar{q}\Gamma^\beta t^B q)\right\}\\ &= \frac{2}{3}(\bar{q}\Gamma^\alpha q)(\bar{q}\Gamma^\beta q) - 2(\bar{q}\Gamma^\alpha t^B q)(\bar{q}\Gamma^\beta t^B q).\end{aligned} \tag{91}$$

One can take another Fierz rearrangement to $(\bar{u}\Gamma^\alpha t^A u)(\bar{u}\Gamma^\alpha t^A u)$ type operators in Eq. (88). Then one can obtain the following relations when taking the symmetric and traceless parts of the operator relations:

$$(\bar{u}\gamma^\alpha t^A u)(\bar{u}\gamma^\beta t^A u)|_{\rm s,t} = -\frac{5}{12}(\bar{u}\gamma^\alpha u)(\bar{u}\gamma^\beta u)|_{\rm s,t} - \frac{1}{4}(\bar{u}\gamma^\alpha\gamma_5 u)(\bar{u}\gamma^\beta\gamma_5 u)|_{\rm s,t} + \frac{1}{4}(\bar{u}\sigma_o{}^\alpha u)(\bar{u}\sigma^{o\beta}u)|_{\rm s,t}, \tag{92}$$

$$(\bar{u}\gamma^\alpha\gamma_5 t^A u)(\bar{u}\gamma^\beta\gamma_5 t^A u)|_{\rm s,t} = -\frac{5}{12}(\bar{u}\gamma^\alpha\gamma_5 u)(\bar{u}\gamma^\beta\gamma_5 u)|_{\rm s,t} - \frac{1}{4}(\bar{u}\gamma^\alpha u)(\bar{u}\gamma^\beta u)|_{\rm s,t} - \frac{1}{4}(\bar{u}\sigma_o{}^\alpha u)(\bar{u}\sigma^{o\beta}u)|_{\rm s,t}, \tag{93}$$

where $|_{\rm s,t}$ means symmetric and traceless. Therefore, only three independent twist-4 (dimension 6 spin-2) matrices remains. Using the twist-4 effects in the deep inelastic scattering data on the proton and neutron target, one can in principle extract two independent constraints to the three independent matrix elements. To determine all the matrix elements, we will additionally use one constraint adopted by Jaffe [47]: $(\bar{u}\sigma_o{}^\alpha u)(\bar{u}\sigma^{o\alpha}u)|_{\rm s,t} = 0$.

2. *Twist-4 operators with mixed quark flavor*

The second type of four-quark operator appearing in the nucleon sum rule are of the following mixed quark flavor operators

$$\begin{aligned}\epsilon_{abc}\epsilon_{a'bc'}\ \gamma^5\gamma^\mu d_c\bar{d}^T_{c'}\gamma^\nu\gamma^5\ (u^T_a C\gamma_\mu \not{q}\gamma_\nu C\bar{u}_{a'}) =&\ \epsilon_{abc}\epsilon_{a'bc'}\ \frac{1}{16}(\gamma^5\gamma^\mu\Gamma_k\gamma^\nu\gamma^5)(\bar{u}_{a'}\Gamma^o u_a)(\bar{d}_{c'}\Gamma^k d_c)\cdot{\rm Tr}\left[\gamma_\mu\not{q}\gamma_\nu C\Gamma_o C\right]\\ \Rightarrow&\ \epsilon_{abc}\epsilon_{a'bc'}\ \frac{1}{16}\Big\{-8q_\alpha(\bar{u}_{a'}\gamma^\alpha u_a)(\bar{d}_{c'}d_c)\\ &-8(q_\beta\gamma_\alpha + g_{\alpha\beta}\not{q})(\bar{u}_{a'}\gamma^\alpha u_a)(\bar{d}_{c'}\gamma^\beta d_c)\\ &+8(q_\beta\gamma_\alpha - g_{\alpha\beta}\not{q})(\bar{u}_{a'}\gamma^\alpha\gamma_5 u_a)(\bar{d}_{c'}\gamma^\beta\gamma_5 d_c)\Big\},\end{aligned} \tag{94}$$

where we have again used Fierz rearrangement to express the operators in terms of quark-anti quark type, and have neglected operators that are odd in parity and time reversal symmetry.

As in the case of the pure quark flavor case, the four-quark condensates in Eq. (94) can be decomposed into two different color structure according to Eq. (91) and Eq. (90). We can not reduce the number of independent operators as in the previous subsubsection because performing a similar Fierz rearrangement as in Eq. (92) and Eq. (93), we find new mixed flavor operators in $(\bar{u}\Gamma^\alpha d)(\bar{d}\Gamma^\beta u)$ type.

3. *Contributions of dimension 6 four-quark to the OPE*

In summary, the independent four-quark condensates appearing in our nucleon sum rule are given in Table I. Not all the matrix elements are known.

As for the dimension 6 spin-0 (scalar) operators, we will assume the factorized form as $\langle\bar{u}u\rangle^2_{\rho,I}$; although this assumption has not been justified. Keeping only the linear density terms, they can be written as

$$\langle[\bar{q}q]_{u,d}\rangle^2_{\rho,I} \Rightarrow \langle\bar{q}q\rangle^2_{\rm vac} + 2f\cdot\left(1 \mp \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)}I\right)\langle\bar{q}q\rangle_{\rm vac}\langle[\bar{q}q]_0\rangle_p\ \rho, \tag{95}$$

where 'f' is a parameter introduced in Ref. [14].

Quark flavor	$q_1 = q_2 = q$	$q_1 \neq q_2$
Dimension 6 spin-2 (Twist-4)	$(\bar{q}\gamma^\alpha\gamma_5 q)(\bar{q}\gamma^\beta\gamma_5 q)\|_{\rm s,t}$	$(\bar{q}_1\gamma^\alpha\gamma_5 q_1)(\bar{q}_2\gamma^\beta\gamma_5 q_2)\|_{\rm s,t}$
	$(\bar{q}\gamma^\alpha q)(\bar{q}\gamma^\beta q)\|_{\rm s,t}$	$(\bar{q}_1\gamma^\alpha q_1)(\bar{q}_2\gamma^\beta q_2)\|_{\rm s,t}$
	$(\bar{q}\sigma_o{}^\alpha q)(\bar{q}\sigma^{o\beta} q)\|_{\rm s,t}$	$(\bar{q}_1\gamma^\alpha\gamma_5 t^A q_1)(\bar{q}_2\gamma^\beta\gamma_5 t^A q_2)\|_{\rm s,t}$
		$(\bar{q}_1\gamma^\alpha t^A q_1)(\bar{q}_2\gamma^\beta t^A q_2)\|_{\rm s,t}$
Dimension 6 spin-1 (Vector)		$(\bar{q}_1\gamma^\alpha q_1)(\bar{q}_2 q_2)$
		$(\bar{q}_1\gamma^\alpha t^A q_1)(\bar{q}_2 t^A q_2)$
Dimension 6 spin-0 (Scalar)	$(\bar{q}\gamma_\alpha\gamma_5 q)(\bar{q}\gamma^\alpha\gamma_5 q)$	$(\bar{q}_1\gamma_\alpha\gamma_5 q_1)(\bar{q}_2\gamma^\alpha\gamma_5 q_2)$
	$(\bar{q}\gamma_\alpha q)(\bar{q}\gamma^\alpha q)$	$(\bar{q}_1\gamma_\alpha q_1)(\bar{q}_2\gamma^\alpha q_2)$
	$(\bar{q}\sigma_{o\alpha} q)(\bar{q}\sigma^{o\alpha} q)$	$(\bar{q}_1\gamma_\alpha\gamma_5 t^A q_1)(\bar{q}_2\gamma^\alpha\gamma_5 t^A q_2)$
		$(\bar{q}_1\gamma_\alpha t^A q_1)(\bar{q}_2\gamma^\alpha t^A q_2)$

TABLE I: Independent four-quark operators appearing in the nucleon OPE with Ioffe's interpolating current. 'q_1' and 'q_2' represents light quark flavors.

Dimension 6 spin-1 (vector) operator are neglected as their contribution to the nuclear symmetry energy is negligible to leading order in density.

Dimension 6 spin-2 are the twist-4 operators. The twist-4 operators appearing in the nucleon sum rule have similar structures as those appearing in the higher twist effects in deep inelastic scattering [34, 40]. If the higher twist effects are measured with precision in DIS for the proton and neutron target, the nucleon expectation value of $(\bar{u}\gamma^\alpha\gamma_5 t^A u)(\bar{d}\gamma^\beta\gamma_5 t^A d)|_{\rm s,t}$ can be estimated with the same precision [33]. With further plausible arguments (Appendix C) on the ratio of u quark and d quark content of the proton such as those used in Eq. (53), one can estimate proton expectation value of $(\bar{u}\gamma^\alpha t^A u)(\bar{u}\gamma^\beta t^A u)|_{\rm s,t}$, $(\bar{u}\gamma^\alpha\gamma_5 t^A u)(\bar{u}\gamma^\beta\gamma_5 t^A u)|_{\rm s,t}$ and $(\bar{u}\gamma^\alpha t^A u)(\bar{d}\gamma^\beta t^A d)|_{\rm s,t}$.

From these condensates, one can estimate nucleon expectation value of all the twist-4 operators for the single flavor case given in the first column in Table I with the extra constraint discussed above. For the mixed flavor condensates given in the second column, one can not deduce all the matrix elements $(\bar{u}\gamma^\alpha\gamma_5 u)(\bar{d}\gamma^\beta\gamma_5 d)|_{\rm s,t}$ and $(\bar{u}\gamma^\alpha u)(\bar{d}\gamma^\beta d)|_{\rm s,t}$ from $(\bar{u}\gamma^\alpha\gamma_5 t^A u)(\bar{d}\gamma^\beta\gamma_5 t^A d)|_{\rm s,t}$ and $(\bar{u}\gamma^\alpha t^A u)(\bar{d}\gamma^\beta t^A d)|_{\rm s,t}$. We will however, neglect $(\bar{u}\gamma^\alpha\gamma_5 u)(\bar{d}\gamma^\beta\gamma_5 d)|_{\rm s,t}$ and $(\bar{u}\gamma^\alpha u)(\bar{d}\gamma^\beta d)|_{\rm s,t}$ in our present analysis, as these mixed quark flavor condensates do not give important contribution to the nuclear symmetry energy in the linear density order.

The proton expectation value of the deducible twist-4 operators can be parameterized into the following forms:

$$\langle(\bar{q}_1\gamma^\alpha\gamma_5 t^A q_1)(\bar{q}_2\gamma^\beta\gamma_5 t^A q_2)\rangle_p|_{\rm s,t} = \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u^\alpha u^\beta - \frac{1}{4}g^{\alpha\beta}\right)T^1_{q_1q_2}, \tag{96}$$

$$\langle(\bar{q}_1\gamma^\alpha t^A q_1)(\bar{q}_2\gamma^\beta t^A q_2)\rangle_p|_{\rm s,t} = \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u^\alpha u^\beta - \frac{1}{4}g^{\alpha\beta}\right)T^2_{q_1q_2}, \tag{97}$$

$$\langle(\bar{q}_1\gamma^\alpha\gamma_5 q_1)(\bar{q}_2\gamma^\beta\gamma_5 q_2)\rangle_p|_{\rm s,t} = \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u^\alpha u^\beta - \frac{1}{4}g^{\alpha\beta}\right)T^3_{q_1q_2}, \tag{98}$$

$$\langle(\bar{q}_1\gamma^\alpha q_1)(\bar{q}_2\gamma^\beta q_2)\rangle_p|_{\rm s,t} = \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u^\alpha u^\beta - \frac{1}{4}g^{\alpha\beta}\right)T^4_{q_1q_2}, \tag{99}$$

where 'q_1' and 'q_2' represent quark flavors. We have extracted the T^is from the matrix elements estimated in Ref. [33] and listed in Table II.

Using the parametrization of the nucleon expectation value of the twist-4 operators together with the linear density approximation given in Eq. (45), the contributions to the correlation function from the four-quark operators can be written as

$$\begin{aligned}
\Pi^E_{D=6,q}(q_0^2,|\vec{q}|) =& -\frac{2}{3q^2}\langle\bar{u}u\rangle^2_{\rho,I} + \frac{1}{q^2}\frac{1}{4\pi\alpha_s}\frac{M_N}{2}[T^1_{ud} - T^2_{ud}]\rho \\
&+ \frac{1}{q^2}\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left([T^1{}_0 - T^2{}_0] - [T^1{}_1 - T^2{}_1]I\right)\rho - \frac{1}{3q^2}\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left([T^3{}_0 - T^4{}_0] - [T^3{}_1 - T^4{}_1]I\right)\rho \\
\Pi^O_{D=6,u}(q_0^2,|\vec{q}|) =& -\frac{4}{q^2}\frac{1}{4\pi\alpha_s}\frac{M_N}{2}[T^1_{ud} - T^2_{ud}]\rho \\
&- \frac{4}{q^2}\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left([T^1{}_0 - T^2{}_0] - [T^1{}_1 - T^2{}_1]I\right)\rho + \frac{4}{3q^2}\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left([T^3{}_0 - T^4{}_0] - [T^3{}_1 - T^4{}_1]I\right)\rho,
\end{aligned} \tag{100}$$

First set (GeV^2)	T^1_{uu}	T^1_{dd}	T^2_{uu}	T^2_{dd}	T^3_{uu}	T^3_{dd}	T^4_{uu}	T^4_{dd}	T^1_{ud}	T^2_{ud}
$K^1_u = K^1_{ud}/\beta$	-0.132	-0.041	0.154	0.048	0.842	0.262	-0.875	-0.272	-0.042	0.049
$K^1_u = K^1_{ud}(\beta+1)/\beta$	-0.071	-0.012	0.070	0.012	0.424	0.072	-0.422	-0.072	-0.042	0.041
$K^1_u = K^1_{ud}$	-0.042	0.002	0.033	-0.002	0.240	-0.012	-0.233	0.012	-0.042	0.031
Second set (GeV^2)	T^1_{uu}	T^1_{dd}	T^2_{uu}	T^2_{dd}	T^3_{uu}	T^3_{dd}	T^4_{uu}	T^4_{dd}	T^1_{ud}	T^2_{ud}
$K^1_u = -K^1_{ud}$	0.215	0.124	-0.432	-0.265	-1.778	-1.091	2.104	1.290	-0.042	0.057
$K^1_u = -K^1_{ud}(\beta+1)/\beta$	0.154	0.100	-0.337	-0.219	-1.336	0.868	1.610	1.046	-0.042	0.056
$K^1_u = -K^1_{ud}/\beta$	0.125	0.085	-0.297	-0.202	-1.137	-0.773	1.395	0.949	-0.042	0.058
$K^1_{ud}/2 = T^1_{ud}$										
$K^i_q = T^i_{qq} + T^i_{ud}$										

TABLE II: Two sets for T^is. Three different classifications of T^i follow classifications of Ref. [33]. Detailed treatment is given in Appendix C.

where ${T^i}_0 = \frac{1}{2}(T^i_{uu} + T^i_{dd})$ and ${T^i}_1 = \frac{1}{2}(T^i_{uu} - T^i_{dd})$, and all the vacuum and scalar condensates are factorized as Eq. (86) and Eq. (95). The corresponding Borel transformations are given as follows:

$$\begin{aligned}\bar{\mathcal{B}}[\Pi_{D=6,q}(q_0^2,|\vec{q}|)] &= \frac{2}{3}\langle\bar{u}u\rangle^2_{\rho,I} L^{\frac{4}{9}} \\ &\quad - \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left\{[T^1_{ud} - T^2_{ud}] - \left([{T^1}_0 - {T^2}_0] - [{T^1}_1 - {T^2}_1]I\right) + \frac{1}{3}\left([{T^3}_0 - {T^4}_0] - [{T^3}_1 - {T^4}_1]I\right)\right\}\rho L^{-\frac{4}{9}}\end{aligned} \tag{101}$$

$$\bar{\mathcal{B}}[\Pi_{D=6,u}(q_0^2,|\vec{q}|)] = \frac{(4\bar{E}_q)}{4\pi\alpha_s}\frac{M_N}{2}\left\{[T^1_{ud} - T^2_{ud}] + \left([{T^1}_0 - {T^2}_0] - [{T^1}_1 - {T^2}_1]I\right) - \frac{1}{3}\left([{T^3}_0 - {T^4}_0] - [{T^3}_1 - {T^4}_1]I\right)\right\}\rho L^{-\frac{4}{9}}. \tag{102}$$

Here, we have neglected the scaling of the matrix elements coming from the anomalous dimension of the dimension 6 operators Γ_{O_n}. Although the twist-4 matrix elements are estimated at the separation scale of 5 GeV, and the matrix element we need is at lower energy scale close to the Borel mass, we will neglect the running of the matrix elements through the anomalous dimension for operator Γ_{O_n}, because the present estimate of the matrix elements already contains $\pm 50\%$ uncertainty. Throughout this paper, we used $\alpha_s \simeq 0.5$ as in Ref. [33, 34].

IV. NUCLEON SUM RULE AND NUCLEAR SYMMETRY ENERGY

We have expressed the self energy contributions of the nucleons that contribute to the nucleon energy as Eq. (16) in terms of the Borel transformed OPE as given in Eqs. (38), (39) and (40). The next step is to substitute Eq. (16) to Eq. (8) to extract the symmetry energy as defined in Eq. (3). Then, there will be the trivial kinematic correction coming from the three momentum dependence in the kinetic energy part of Eq. (16). This term is universal and corresponds to the term in Eq. (6). Instead of following the full procedure, in this work, we will just concentrate on the contribution coming from the scalar and vector self energy. This corresponds to calculating the contribution to the nuclear symmetry energy from potentials in effective models.

A. QCD sum rule formula

The quasi-nucleon self energies in rest frame can be obtained in QCD sum rules by taking the ratios Eq. (38)/Eq. (39) and Eq. (40)/Eq. (39) for both the proton and neutron:

$$\begin{aligned}E_{q,V(I)} \equiv \Sigma_v + M_N^* &= \frac{\mathcal{N}^{n,p}(\rho)}{\mathcal{D}^{n,p}(\rho)} \\ &= \frac{\bar{\mathcal{B}}[\Pi_s^{n,p}(q_0^2,|\vec{q}|)] + \bar{\mathcal{B}}[\Pi_u^{n,p}(q_0^2,|\vec{q}|)]}{\bar{\mathcal{B}}[\Pi_q^{n,p}(q_0^2,|\vec{q}|)]},\end{aligned} \tag{103}$$

where subscripts $q, V(I)$ are meant to represent the potential part of Eq. (16) in the asymmetric nuclear matter. To discuss different approximations of self energies in terms of the density and the asymmetric factor, we intro-

duce the following symbols $\mathcal{N}^{n,p}_{(\rho^m,I^l)}(\rho)$ and $\mathcal{D}^{n,p}_{(\rho^m,I^l)}(\rho)$:

$$\mathcal{N}^{n,p}(\rho) = \mathcal{N}^{n,p}_{(\rho^0,I^0)} + \mathcal{N}^{n,p}_{(\rho,I^0)}\rho + \left[\mathcal{N}^{n,p}_{(\rho,I)}\rho\right] I + \sum_2^m \sum_2^l \left[\mathcal{N}^{n,p}_{(\rho^m,I^l)}\rho^m\right] I^l, \tag{104}$$

$$\mathcal{D}^{n,p}(\rho) = \mathcal{D}^{n,p}_{(\rho^0,I^0)} + \mathcal{D}^{n,p}_{(\rho,I^0)}\rho + \left[\mathcal{D}^{n,p}_{(\rho,I)}\rho\right] I + \sum_2^m \sum_2^l \left[\mathcal{D}^{n,p}_{(\rho^m,I^l)}\rho^m\right] I^l, \tag{105}$$

where the superscript 'n', 'p' represents either the neutron or the proton respectively. For the pair of subscripts (ρ^m, I^l), the first index represents the order of the density, while the the second index represents the isospin. Due to isospin symmetry, the iso-scalar terms have the following relations:

$$\mathcal{N}^n_{(\rho^m,I^l)} = (-1)^l \mathcal{N}^p_{(\rho^m,I^l)}, \tag{106}$$

$$\mathcal{D}^n_{(\rho^m,I^l)} = (-1)^l \mathcal{D}^p_{(\rho^m,I^l)}, \tag{107}$$

where l is the integer for the order of the isospin. All the terms appearing are summarized in the Appendix D.

Because the dominant term of $\mathcal{D}^{n,p}(\rho)$ is $\mathcal{D}^{n,p}_{(\rho^0,I^0)}$, one can expand the denominator in terms of $(1/\mathcal{D}^{n,p}_{(\rho^0,I^0)})$ times condensate. After rewriting this with powers of ρ and I, one can express the potential part of a single nucleon energy as

$$E^{n,p}_V(\rho,I) = E^{n,p}_{V,(\rho^0,I^0)} + \sum_{k=1}^{\infty}\sum_{i=0}^{\infty}\left(\left[E^{n,p}_{V,(\rho^k,I^i)}\rho^k\right] I^i\right), \tag{108}$$

where $E^{n,p}_{V,(\rho^k,I^i)}$ are written in terms of $\mathcal{N}^{n,p}(\rho)$ and $\mathcal{D}^{n,p}(\rho)$. Averaging Eq. (108) as Eq. (8) and collecting terms of I^2 from Eq. (2), one can extract $E^{\rm sym}_V(\rho)$ as follows:

$$\begin{aligned} E^{\rm sym}_V(\rho) =& \frac{1}{2}\left[\frac{1}{2}\rho\cdot(E^n_{V,(\rho,I)} - E^p_{V,(\rho,I)}) + \frac{1}{3}\rho^2\cdot(E^n_{V,(\rho^2,I)} - E^p_{V,(\rho^2,I)}) + \frac{1}{4}\rho^3\cdot(E^n_{V,(\rho^3,I)} - E^p_{V,(\rho^3,I)}) + \cdots\right] \\ &+ \frac{1}{2}\left[\frac{1}{3}\rho^2\cdot(E^n_{V,(\rho^2,I^2)} + E^p_{V,(\rho^2,I^2)}) + \frac{1}{4}\rho^3\cdot(E^n_{V,(\rho^3,I^2)} + E^p_{V,(\rho^3,I^2)}) + \cdots\right]. \end{aligned} \tag{109}$$

For terms linear in density, one can see that the first term in the upper bracket of Eq. (109) corresponds to the form given in Eq. (9). The explicit expression in terms of $\mathcal{N}^n_{(\rho^m,I^l)}$ and $\mathcal{D}^n_{(\rho^m,I^l)}$ is

$$E^{\rm sym}_{V,\rho} = \frac{1}{4}\rho\cdot\left[\frac{1}{\mathcal{D}^p_{(\rho^0,I^0)}}\cdot\left(-2\mathcal{N}^p_{(\rho,I)}\right) - \frac{\mathcal{N}^p_{(\rho^0,I^0)}}{(\mathcal{D}^p_{(\rho^0,I^0)})^2}\cdot\left(-2\mathcal{D}^p_{(\rho,I)}\right)\right], \tag{110}$$

valid to leading order in density.

When higher density dependence of the condensates are calculated, Eq. (109) provides a systematic expression of $E^{\rm sym}_V(\rho)$ that includes higher $\rho^{n\geq 2}$ terms.

B. Sum rule analysis

In principle, a physical quantity extracted from the QCD sum rule should not depend on the Borel parameter M^2. However, since we truncate the OPE at finite mass dimension, such a physical quantity should be obtained within a reliable range of M^2 (Borel window) with "plateau". While we do not find the most stable "plateau" with a extremum in the appropriate Borel window, one finds that the results has only a weak dependence on M^2.

The well accepted Borel window for nucleon sum rule is 0.8 GeV$^2 \leq M^2 \leq$ 1.4 GeV^2 [24]. But as our sum rule contains the newly added twist-4 four-quark operators, the Borel window needs to be re-examined. We determine the upper Borel window by requiring that the quasi-nucleon contribution to be more than 50% of the total sum rule so that the continuum contribution be less than 50%. As for the lower limit, for the same OPE, we restrict the contribution from the highest mass dimension operator to be less than 50% of the total contribution. For the quasi-nucleon energy in medium rest frame, we applied this prescription to the R.H.S of Eqs. (38), (39) and (40).

The Borel curves for the three invariants (Eqs. (38)), (39) and (40)) are plotted in Fig. 1. Here, all the graphs are obtained with the T^is using the $K^1_u = K^1_{ud}(\beta+1)/\beta$ estimates from the first set of Table II. From the left figure of Fig. 1, one can get an acceptable Borel window

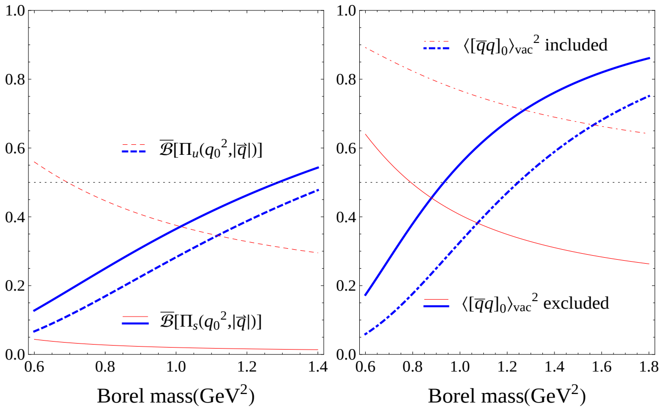

FIG. 1: Borel window for three invariants. Left : the borel window for $\bar{\mathcal{B}}[\Pi_s(q_0^2,|\vec{q}|)]$ and $\bar{\mathcal{B}}[\Pi_u(q_0^2,|\vec{q}|)]$. Right : the borel window for $\bar{\mathcal{B}}[\Pi_q(q_0^2,|\vec{q}|)]$. In both figure, the thick lines increasing with the Borel mass represent the ratio (the contribution of highest dimensional operators)/(the total OPE), and the thin lines decreasing with the Borel mass represent the ratio (the continuum contribution)/(total contribution). These graphs are obtained with T^is in $K_u^1 = K_{ud}^1(\beta+1)/\beta$ estimation from the first set of Table II.

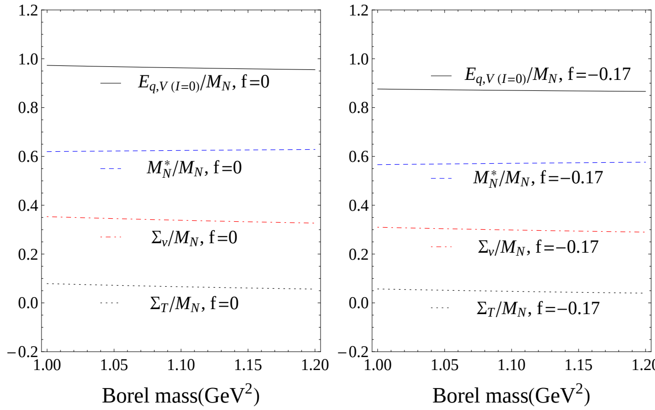

FIG. 2: The ratios between quasi-nucleon self energies and the vacuum mass. The different lines represent (M_N^*/M_N (dashed blue), Σ_v/M_N (dot-dashed red), Σ_T/M_N (dotted black) and $E_{q,V(I=0)}/M_N$ (solid black)) respectively.

for $\bar{\mathcal{B}}[\Pi_s(q_0^2,|\vec{q}|)]$ (Eq. (38)) and $\bar{\mathcal{B}}[\Pi_u(q_0^2,|\vec{q}|)]$ (Eq. (40)). However, in the right figure of Fig. 1, $\bar{\mathcal{B}}[\Pi_q(q_0^2,|\vec{q}|)]$ do not provide an acceptable Borel window. While the usual Borel window is obtained by requiring that the power and continuum correction are both less than 50% of the total OPE, we will loosen the condition to be less than 75% in this case.

This large power correction, may be caused by an overestimated $\langle[\bar{q}q]_0\rangle_{\rm vac}^2$. As mentioned in the previous section, all the vacuum expectation values of four-quark operators are factorized as in Eq. (95). Only large N_c supports factorization in the vacuum. Hence, the generalization to the nuclear medium can only be an order of magnitude estimate with large uncertainty. For example, as one can see in the right figure of Fig. 1, the lower and upper boundary from $\bar{\mathcal{B}}[\Pi_q(q_0^2,|\vec{q}|)]$ are already largely affected by whether the vacuum value $\langle[\bar{q}q]_0\rangle_{\rm vac}^2$ is included or not. Also, the neglected twist-4 matrix elements T_{ud}^3 and T_{ud}^4 for $(\bar{u}\gamma^\alpha\gamma_5 u)(\bar{d}\gamma^\alpha\gamma_5 d)|_{\rm s,t}$ and $(\bar{u}\gamma^\alpha u)(\bar{d}\gamma^\alpha d)|_{\rm s,t}$ may be another reason for the large uncertainty. If the vacuum expectation value of four-quark operators and T_{ud}^3 and T_{ud}^4 can be determined well, we can discuss about the stability of our sum rule in a more reliable way. The second set of T^is from Table II do not produce any acceptable Borel window. In conclusion, we will use the results from the following Borel window: $1.0\ {\rm GeV}^2 \leq M^2 \leq 1.2\ {\rm GeV}^2$.

In the analysis to follow, for the symmetric nuclear matter case, we will denote the twist-4 condensates contribution to the quasi-nucleon self energy as Σ_T and the total quasi-nucleon self energy in the rest frame as $E_{q,V(I=0)}$. For the asymmetric nuclear matter case, we will use two sum rules for $E_V^{\rm sym}$: one that includes contributions up to order ρ terms and another one up to ρ^2. The former sum rule will be called the linear ρ sum rule ($E_{V,\rho}^{\rm sym}$) and the later ρ^2 sum rule ($E_{V,\rho^2}^{\rm sym}$). As for the value for the anti-nucleon pole, an optimal 'in-medium' value ranged -0.2 GeV $\leq \bar{E}_q \leq -0.4$ GeV will be used for each different estimation of twist-4 matrix elements in the sum rule for the quasi-nucleon self energy, while the 'bare' value $\bar{E}_q = -M_N$ will be used in the sum rule for the nuclear symmetry energy. This is so because the quasi-hole contribution in the nuclear symmetry energy comes with a term proportional to the density (Eq. (8)). Nuclear matter density ρ is set at the saturation density $\rho_0 = 0.16\ {\rm fm}^{-3}$ and the corresponding quasi-nucleon three-momentum $|\vec{q}|$ is taken to be 270 MeV, the Fermi momentum of normal nucleus ($\rho_0 = 0.16\ {\rm fm}^{-3}$). The light quark ($u$, d quark) mass m_q is taken to be 5 MeV.

As for the density dependence of dimension 6 spin-0 condensates, different f values are used for every estimations of twist-4 matrix elements. For the first set of Table II; $f = -0.2$ for $K_u^1 = K_{ud}^1/\beta$ (corresponding $\bar{E}_q = -0.26$ GeV), $f = -0.17$ (corresponding $\bar{E}_q = -0.34$ GeV)for $K_u^1 = K_{ud}^1(\beta+1)/\beta$ and $f = -0.12$ for $K_u^1 = K_{ud}^1$ (corresponding $\bar{E}_q = -0.4$ GeV). This parameter set of fs are chosen to satisfy the self consistency constraint as given in Eq. (17) for the quasi-hole value. Again, the second set of the Table II do not provide a set of fs which satisfies the constraint Eq. (17). Detailed discussion for related parameters (f and $\bar{E}_q$) will be given in later section.

1. *Symmetric nuclear matter*

First, we investigate the quasi-nucleon self energies in the symmetric nuclear matter with twist-4 condensates. Throughout the analysis, we check the result against the $I = 0$ case. In Fig. 2, we plot the ratio to the nucleon mass in vacuum of the in-medium scalar self energy (M_N^*/M_N), the vector self energy (Σ_v/M_N),

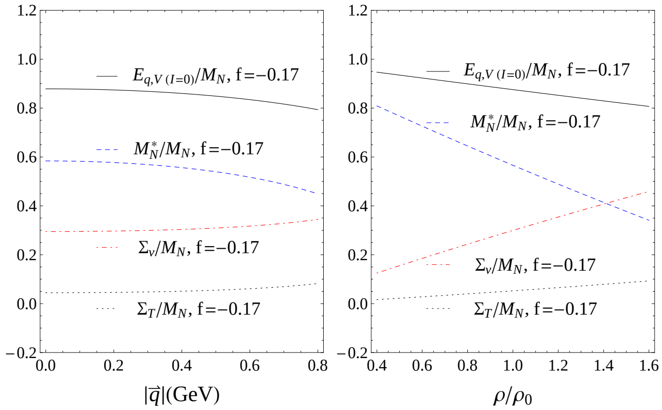

FIG. 3: Density (right) and $|\vec{q}|$ (left) dependence of the ratios between quasi-nucleon self energies and the vacuum mass.

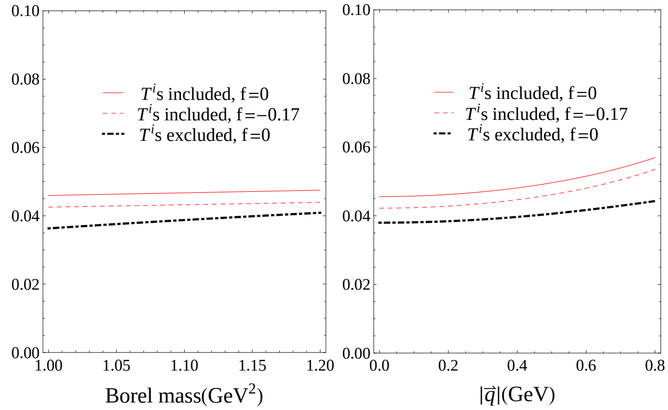

FIG. 4: $E^{\rm sym}_{V,\rho}$ (left) and its $|\vec{q}|$ dependence (right). The unit of the vertical axis is GeV.

the twist-4 condensates contribution (Σ_T/M_N) and the potential part of total quasi-nucleon self energy in rest frame ($E_{q,V(I=0)}/M_N$). For the twist-4 matrix elements, we take $K^1_u = K^1_{ud}(\beta+1)/\beta$ from the first set of Table II and corresponding $\bar{E}_q = -0.34$ GeV, which gives the average result. From our analysis, we find that the contribution of the the twist-4 condensates give enhancement of the quasi-nucleon self energy by $\sim$ 50 MeV. When $f = 0$, we find the ratio $E_{q,V(I=0)}/M_N \simeq 0.96$, $M^*_N/M_N \simeq 0.62$ and $\Sigma_v/M_N \simeq 0.33$. By using aforementioned parameter set for $f < 0$ and $\bar{E}_q$, the ratios become $E_{q,V(I=0)}/M_N \simeq 0.87$, $M^*_N/M_N \simeq 0.56$ and $\Sigma_v/M_N \simeq 0.30$, which are comparable with previous studies [11–13]. When the second set of Table II is used for the twist-4 matrix elements,we do not find a stable behavior in the same Borel window in contrast to the case with the first set as shown previously. By setting $f > 0$ for the second set, $E_{q,V(I=0)}/M_N$ can be adjusted to ~ 0.9 which is a typically acceptable value. But even so, there is no reasonable f and $\bar{E}_q$ for M^*_N/M_N and Σ_v/M_N which satisfies Eq. (17). The estimates for T^is given in the second set of Table II do not reproduce the aspect of the nucleon sum rule that is consistent with the Driac Phenomenology[11]. Hence, we will continue the present analysis with estimates for T^is given by the the first set of Table II.

The quasi-nucleon three momentum dependence is plotted in the left figure of Fig. 3 for $f < 0$ cases: one finds that the ratio Σ_v/M_N and Σ_T/M_N do not depend strongly on the quasi-nucleon three momentum. On the other hand, M^*_N/M_N shows significant change when $|\vec{q}| \geq 0.5$ GeV as in Ref. [14]. So this sum rule analysis works in $0 \leq |\vec{q}| \leq 0.5$ GeV region, which is consistent with our phenomenological ansatz that assumes a momentum independent self energy.

As all the condensates in our nucleon sum rule are estimated up to linear order in density, the results may be valid at least near the nuclear saturation density region. In the right figure of Fig. 3, the density dependence of the quasi-nucleon self energies is plotted for $0.4 \leq \rho/\rho_0 \leq 1.6$. Here we used the parameter set $f = -0.17$ and $\bar{E}_q = -0.34$ GeV determined at the saturation density, as our nucleon sum rule do not depend strongly on $\bar{E}_q$ in -0.6 GeV $\leq \bar{E}_q \leq -0.3$ GeV which covers naively estimated $\bar{E}_q$ in $0.4 \leq \rho/\rho_0 \leq 1.6$. Σ_v/M_N and Σ_T/M_N are enhanced as the nuclear matter becomes denser but M^*_N/M_N is reduced.

2. Asymmetric nuclear matter

In our sum rule, nuclear bulk properties in the asymmetric nuclear matter are parameterized by the asymmetry factor I. If one plots the quasi-nucleon self energy as a function of I to leading order in density, $E^{\rm sym}_{V,\rho}$ can be obtained from the difference between the slope of the quasi-neutron and the quasi-proton (Eq. (110)).

$E^{\rm sym}_{V,\rho}$ is plotted in Fig. 4. One finds that $E^{\rm sym}_{V,\rho}$ ranges from 40 to 50 MeV, which agrees with previous studies. The results in the figure also show that including the twist-4 contribution slightly enhances the nuclear symmetry energy.

In the right figure of Fig. 4, one finds that $E^{\rm sym}_{V,\rho}$ do not depend strongly on the quasi-nucleon three momentum up to 0.5 GeV. This result agrees with the quasi-nucleon three momentum dependence of the quasi-nucleon self energy. When the second set of Table II are used for the T^is, we find that $E^{\rm sym}_{V,\rho}$ depends strongly on the quasi-nucleon three momentum than the first set.

One can also work out $E^{\rm sym}_{V,\rho^2}$, although with larger uncertainty than that of $E^{\rm sym}_{V,\rho}$. The density dependence of $E^{\rm sym}_{V,\rho^2}$ and $E^{\rm sym}_{K}$ in $0.4 \leq \rho/\rho_0 \leq 1.6$ are plotted in Fig. 5. Here again, the four-quark condensates give a important change to the density behavior of $E^{\rm sym}_{V}$. For $f = 0$, the contribution of T^is gives slight enhancement of $E^{\rm sym}_{V,\rho^2}$ at higher nuclear matter density but for $f = -0.17$, it gives reduction of $E^{\rm sym}_{V,\rho^2}$ at higher density. This means, in the symmetry energy case also, the scalar four-quark operators in our sum rule may play a important role in reduc-

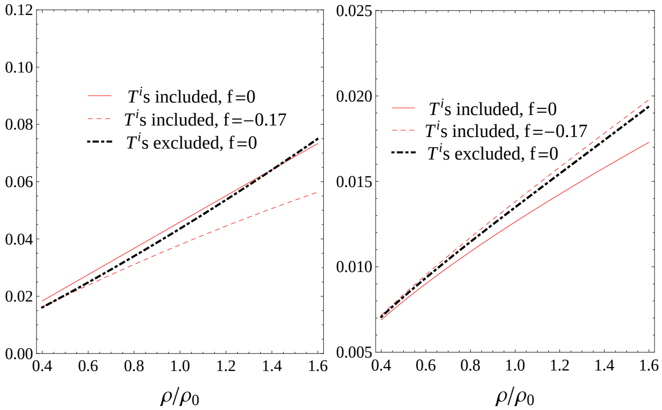

FIG. 5: Density dependence of $E^{\rm sym}$. Left : density dependence of $E^{\rm sym}_{V,\rho^2}$. Right : density dependence of $E^{\rm sym}_K$. The unit of the vertical axis is GeV.

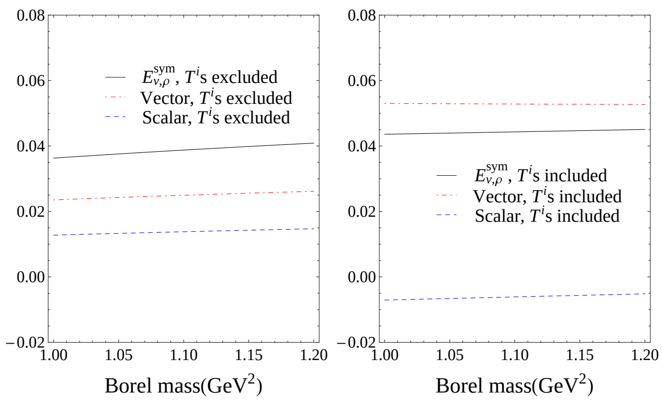

FIG. 6: Scalar-vector self energy decomposition of $E^{\rm sym}_{V,\rho}$. Left : $E^{\rm sym}_{V,\rho}$ without twist-4 contribution. Right : $E^{\rm sym}_V$ with twist-4 contribution. The unit of the vertical axis is GeV.

tion mechanism of the nuclear symmetry energy. However, $E^{\rm sym}_K$ is slightly reduced by T^is as twist-4 matrix elements enhances M^*_N/M_N. The parameter set $f<0$ contributes differently $E^{\rm sym}_V$ and $E^{\rm sym}_K$; reduction for $E^{\rm sym}_V$ and enhancement for $E^{\rm sym}_K$ in $0.4\le\rho/\rho_0\le 1.6$.

In Fig. 6, we plot the scalar $\left(\frac{\bar{\mathcal{B}}[\Pi_s(q_0^2,|\vec{q}|)]}{\bar{\mathcal{B}}[\Pi_q(q_0^2,|\vec{q}|)]}\right)$ and vector $\left(\frac{\bar{\mathcal{B}}[\Pi_u(q_0^2,|\vec{q}|)]}{\bar{\mathcal{B}}[\Pi_q(q_0^2,|\vec{q}|)]}\right)$ self energy part of $E^{\rm sym}_V$. In the left figure, we plot the result without the twist-4 contribution, while in the right hand figure, we include the contribution from twist-4 matrix elements. While both the scalar and vector self energy give positive contribution to the self energy in the left hand figure, one finds that in the right hand side, the scalar and vector give negative and positive contributions respectively. The right figure is consistent with the general trends in RMFT results [29], which shows scalar self energy part gives negative contribution and vector self energy part gives positive contribution from the exchange of δ and ρ meson exchanges respectively. One can infer from this result that the twist-4 contribution mimics the exchange of the δ and ρ meson

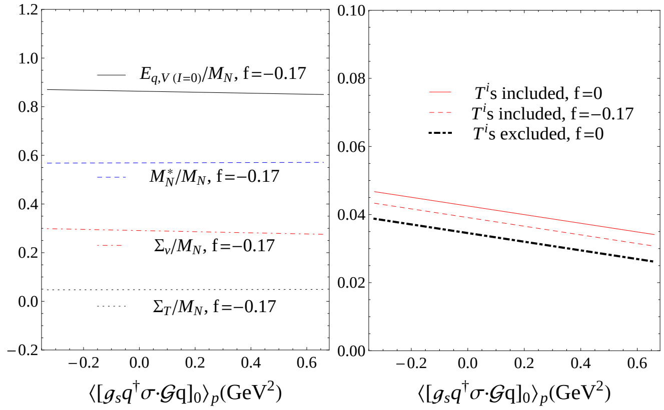

FIG. 7: Sensitivity analysis with $\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_0\rangle_p$. Left : $\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_0\rangle_p$ dependence of $E_{q,V(I=0)}$. Right : $\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_0\rangle_p$ dependence of $E^{\rm sym}_{V,\rho}$. The unit of the vertical axis for the right figure is GeV.

and that it constitutes an essential part in the origin of the nuclear symmetry energy from QCD.

3. *Uncertainties*

In general, there are two quark-gluon mixed operators with contracted spin indices, $\langle[g_s\bar{q}\sigma\cdot\mathcal{G}q]_0\rangle_p$ and $\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_0\rangle_p$, which are not accurately determined. $\langle[g_s\bar{q}\sigma\cdot\mathcal{G}q]_0\rangle_p$ does not appear in our sum rule with Ioffe's nucleon interpolating field (Eq. (19)). As for the operator $\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_0\rangle_p$, the proton expectation value has been estimated in Refs. [13, 14, 45, 46] to be in the range of -0.33 GeV$^2\le\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_0\rangle_p\le 0.66$ GeV^2. Hence, we investigate the $\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_0\rangle_p$ dependence in Fig. 7. The matrix element $\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_0\rangle_p$ do not give important contribution to the quasi-nucleon self energies in the range -0.33 GeV$^2\le\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_0\rangle_p\le 0.66$ GeV^2 as we are not interested in the accuracy of 10 MeV. However, for $E^{\rm sym}_{V,\rho}$, $\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_0\rangle_p$ such magnitude gives nontrivial fractional change to $E^{\rm sym}_{V,\rho}$. We choose the value as $\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_0\rangle_p=-0.33$ GeV^2 in this study as was done in Ref. [13, 14].

C. Conclusion

In this paper we studied the nuclear bulk properties in the asymmetric nuclear matter by calculating the quasi-nucleon self energies with QCD sum rule approach. In particular, we identified all the twist-4 local condensates appearing in the nucleon sum rule. Using the existing estimates for the twist-4 matrix element from DIS, we were able to find the magnitudes of all the twist-4 matrix elements (T^i) in our sum rule except for two mixed quark flavor type condensates. We have calculated the nuclear symmetry energy and found that twist-4 contributions

are non-negligible and essential to give a phenomenologically consistent result with RMFT for the quasi-nucleon self energy and the nuclear symmetry energy.

For the symmetric nuclear matter case, we found that $E_{q,V(I=0)}$ is enhanced by ~ 50 MeV with T^is in the first set of Table II. As the T^is in the first set of Table II provides qualitatively reliable sum rule results while T^is in the second set of Table II do not, we conclude that taking the sum rule results with T^is in the first set is reasonable choice. With parameter set $f<0$, dimension 6 spin-0 (scalar) operators reduces $E_{q,V(I=0)}/M_N$ to ~ 0.87.

For the asymmetric nuclear matter case, we confirmed two meaningful facts. First, QCD sum rule technique can be used to successfully reproduce the acceptable result for the nuclear symmetry energy at the nuclear matter density. Second, dimension 6 spin-2 (twist-4) condensates plays an essential role in making the scalar part contribute negatively to the self energy and thus providing a consistent picture for the $E^{\rm sym}$ with the RMFT results [29]:

$$E_V^{\rm sym} = \frac{1}{2}\left[f_\rho - f_\delta\left(\frac{m^*}{E_F^*}\right)\right]\rho_B, \tag{111}$$

where f_ρ is isovector ρ meson coupling, f_δ is isoscalar δ (f_0) coupling and ρ_B is the nuclear matter density. Moreover, our approach provides a first attempt to understanding the origin of $E_{V,\rho}^{\rm sym}$ in terms of local operators directly from QCD. This extends the analogy between QCD sum rules to RMFT for the symmetric nuclear matter established in Refs. [11, 13, 14], to the asymmetric limit.

While the uncertainties in T^is and four-quark scalar with parametrization f are still large, attempts to measure the twist-4 contribution in DIS at the future upgrade at Jefferson Lab is expected to lower the uncertainties and provide more insights to the value for the nuclear expectation value of four-quark operators.

Acknowledgments

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Appendix A: Baryon octet mass relation

In this section, we summarize an essential argument for obtaining $\langle[\bar{q}q]_1\rangle_p = \frac{1}{2}\left(\langle p|\bar{u}u|p\rangle - \langle p|\bar{d}d|p\rangle\right)$ from Ref. [22] and Ref. [25]. Phenomenologically, the nucleon mass can be expressed in terms of the matrix element of the trace of the energy-momentum tensor:

$$m_N\langle N|\bar{\psi}_N\psi_N|N\rangle = \langle N|\theta^\mu{}_\mu|N\rangle. \tag{A1}$$

Using the equations of motion, the trace of the energy-momentum tensor can be written as

$$\begin{aligned}\theta^\mu{}_\mu &= m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s + \sum_{h=c,t,b} m_h\bar{h}h + \cdots \\ &= \left(\frac{\bar{\beta}}{4\alpha_s}\right)\mathcal{G}^2 + m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s + O(\mu^2/4m_h^2),\end{aligned} \tag{A2}$$

where h are heavy quark fields, and the gluonic term comes from the trace anomaly [41–43]. $\bar{\beta} = -9\alpha_s^2/2\pi$ is the "reduced" Gellmann-Low function in which heavy quark contribution has been subtracted using the heavy quark expansion [44].

Eq. (A2) can be applied to the lower-lying baryon octet. The lower-lying baryon octet mass relation with SU(3) flavor symmetry is as follows:

$$\begin{aligned} m_p &= A + m_uB_u + m_dB_d + m_sB_s, \\ m_n &= A + m_uB_d + m_dB_u + m_sB_s, \\ m_{\Sigma^+} &= A + m_uB_u + m_dB_s + m_sB_d, \\ m_{\Sigma^-} &= A + m_uB_s + m_dB_u + m_sB_d, \\ m_{\Xi^0} &= A + m_uB_d + m_dB_s + m_sB_u, \\ m_{\Xi^-} &= A + m_uB_s + m_dB_d + m_sB_u, \end{aligned} \tag{A3}$$

where $A \equiv \langle(\bar{\beta}/4\alpha_s)G^2\rangle_p$, $B_u \equiv \langle\bar{u}u\rangle_p$, $B_d \equiv \langle\bar{d}d\rangle_p$ and $B_s \equiv \langle\bar{s}s\rangle_p$. In this relation, correction terms for hyperon is neglected [28]. From (A3) one can obtain

$$\langle p|\bar{u}u|p\rangle - \langle p|\bar{d}d|p\rangle = \frac{(m_{\Xi^0} + m_{\Xi^-}) - (m_{\Sigma^+} + m_{\Sigma^-})}{2m_s - (m_u + m_d)}. \tag{A4}$$

Appendix B: A simple constraint for twist-4 operators from zero identity

In this section, we show an explicit calculation for a simple constraint using the zero identity [35]. For pure quark flavor diquark structure,

$$\epsilon_{abc}(u_a^T C\Gamma u_b) = 0 \qquad \text{If } (C\Gamma)^T = -C\Gamma, \tag{B1}$$

where Γ is Clifford basis. So ($\Gamma = \{I, \gamma_5, i\gamma_\mu\gamma_5\}$) satisfies the above condition. Therefore, a constraint for four-quark operator can be obtained by requiring that the Fierz transformed form of the products of above diquarks are zero. An example is the following:

$$\begin{aligned}
\epsilon_{abc}\epsilon_{a'b'c}(u_a^T C\gamma_\mu\gamma_5 u_b)(\bar{u}_{b'}^T\gamma_\nu\gamma_5 C\bar{u}_{a'}) &= \epsilon_{abc}\epsilon_{a'b'c}\,\frac{1}{16}\,(\bar{u}_{a'}\Gamma^o u_a)(\bar{u}_{b'}\Gamma^k u_b)\cdot \mathrm{Tr}\left[\gamma_\mu\Gamma_k\gamma_\nu C\Gamma_o^T C\right] \\
&= \epsilon_{abc}\epsilon_{a'b'c}\,\frac{1}{16}\,\Big\{(\bar{u}_{a'}u_a)(\bar{u}_{b'}u_b)\cdot(4g_{\mu\nu}) \\
&\quad + (\bar{u}_{a'}\gamma_5 u_a)(\bar{u}_{b'}\gamma_5 u_b)\cdot(-4g_{\mu\nu}) \\
&\quad + (\bar{u}_{a'}\gamma^\alpha u_a)(\bar{u}_{b'}\gamma^\beta u_b)\cdot(4S_{\mu\beta\nu\alpha}) \\
&\quad - (\bar{u}_{a'}\gamma^\alpha\gamma_5 u_a)(\bar{u}_{b'}\gamma^\beta\gamma_5 u_b)\cdot(4S_{\mu\beta\nu\alpha}) \\
&\quad - (\bar{u}_{a'}\sigma^{\alpha\bar{\alpha}}u_a)(\bar{u}_{b'}\sigma^{\beta\bar{\beta}}u_b)\cdot\frac{1}{4}\mathrm{Tr}\left[\gamma_\mu\sigma_{\beta\bar{\beta}}\gamma_\nu\sigma_{\alpha\bar{\alpha}}\right] \\
&\quad + (\bar{u}_{a'}\gamma^\alpha u_a)(\bar{u}_{b'}\gamma^\beta\gamma_5 u_b)\cdot(8i\epsilon_{\mu\alpha\nu\alpha}) \\
&\quad + (\bar{u}_{a'}u_a)(\bar{u}_{b'}\sigma^{\alpha\bar{\alpha}}u_b)\cdot(8ig_{\alpha\mu}g_{\bar{\alpha}\nu}) \\
&\quad + (\bar{u}_{a'}\gamma_5 u_a)(\bar{u}_{b'}\sigma^{\alpha\bar{\alpha}}u_b)\cdot(4\epsilon_{\mu\nu\alpha\bar{\alpha}})\Big\} = 0,
\end{aligned} \tag{B2}$$

where $S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\alpha\nu} - g_{\mu\nu}g_{\alpha\beta}$. By subtracting Eq. (B2) from Eq. (87), Eq. (87) can be simplified into Eq. (88).

Appendix C: Estimation of twist-4 matrix elements

In this section, we provide a detailed treatment for extracting T^is from the values estimated in Ref. [33]. In Ref. [33], twist-4 operators which appear in our nucleon sum rule are given as

$$\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u_\alpha u_\beta - \frac{1}{4}g_{\alpha\beta}\right)K_u^1 = \langle(\bar{u}\gamma_\alpha\gamma_5 t^A u)(\bar{u}\gamma_\beta\gamma_5 t^A u)\rangle_p|_{\mathrm{s,t}} + \langle(\bar{u}\gamma_\alpha\gamma_5 t^A u)(\bar{d}\gamma_\beta\gamma_5 t^A d)\rangle_p|_{\mathrm{s,t}}, \tag{C1}$$

$$\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u_\alpha u_\beta - \frac{1}{4}g_{\alpha\beta}\right)K_u^2 = \langle(\bar{u}\gamma_\alpha t^A u)(\bar{u}\gamma_\beta t^A u)\rangle_p|_{\mathrm{s,t}} + \langle(\bar{u}\gamma_\alpha t^A u)(\bar{d}\gamma_\beta t^A d)\rangle_p|_{\mathrm{s,t}}, \tag{C2}$$

$$\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u_\alpha u_\beta - \frac{1}{4}g_{\alpha\beta}\right)K_{ud}^1 = 2\langle(\bar{u}\gamma_\alpha\gamma_5 t^A u)(\bar{d}\gamma_\beta\gamma_5 t^A d)\rangle_p|_{\mathrm{s,t}}, \tag{C3}$$

where we changed the normalization for the nucleon state appearing in Ref. [33] to the following:

$$\langle N(p)|N(p')\rangle = \frac{\omega_p}{M_N}(2\pi)^3\delta^3(\vec{p}-\vec{p'}), \tag{C4}$$

with $\omega_p = p_0 = \sqrt{\vec{p}^2 + M_N^2}$. Here only K_{ud}^1 is uniquely determined: $K_{ud}^1 = -0.083\ \mathrm{GeV}^2$. One can set a constraint $|K_d^1| = |K_u^1|\cdot\beta < |K_{ud}^1| < |K_u^1|$ with an ansatz that the ratio K_d^i/K_u^i is equal to the momentum fraction of the d and u quarks in the nucleon:

$$K_d^i/K_u^i \simeq \frac{\int x(d(x)+\bar{d}(x))dx}{\int x(u(x)+\bar{u}(x))dx} \equiv \beta = 0.476. \tag{C5}$$

Varying K_u^1 in the constraint above, one can estimate K_u^2 as a functions of K_u^1 (Table III).

K_u^1	K_u^2	K_u^1	K_u^2
$K_{ud}^1/\beta = -0.173$	0.203	$-K_{ud}^1 = 0.083$	-0.181
$K_{ud}^1(\beta+1)/2\beta = -0.112$	0.110	$-K_{ud}^1(\beta+1)/2\beta = 0.112$	-0.225
$K_{ud}^1 = -0.083$	0.066	$-K_{ud}^1/\beta = 0.173$	-0.318
(GeV^2)			

TABLE III: Table for K_u^i from Ref. [33].

T_{uu}^1 and T_{dd}^1 can be easily estimated by taking $T_{ud}^1 = \frac{1}{2}K_{ud}^1$ from K_u^1 and K_d^1:

$$T_{uu}^1 = K_u^1 - T_{ud}^1, \tag{C6}$$

$$T_{dd}^1 = K_d^1 - T_{ud}^1, \tag{C7}$$

$$T_{ud}^1 = \frac{1}{2}K_{ud}^1 = -0.042\ \text{GeV}^2. \tag{C8}$$

Similarly, one can try to obtain T_{uu}^2 and T_{dd}^2 from K_u^2 and K_d^2. As $T_{ud}^2 = \frac{1}{2}K_{ud}^2$ has not been determined uniquely as $T_{ud}^1 = \frac{1}{2}K_{ud}^1$, we assumed that the ratio T_{uu}^1/T_{dd}^1 is equal to T_{uu}^2/T_{dd}^2. Then, by the following relation, one can estimate T_{qq}^2s:

$$T_{uu}^2 = (K_u^2 - K_d^2)\cdot\left(1 - \frac{T_{dd}^1}{T_{uu}^1}\right)^{-1}, \tag{C9}$$

$$T_{dd}^2 = \left(\frac{T_{dd}^1}{T_{uu}^1}\right)\cdot T_{uu}^2, \tag{C10}$$

$$T_{ud}^2 = \frac{1}{2}\left([K_u^2 + K_d^2] - [T_{uu}^2 + T_{dd}^2]\right), \tag{C11}$$

where $K_u^2 - K_d^2$ and $K_u^2 + K_d^2$ can be obtained from Table III.

For pure quark flavor case, T_{qq}^3 and T_{qq}^4 can be obtained from Eq. (92) and Eq. (93). As discussed in Ref. [47], we neglect $(\bar{u}\sigma_o{}^{\alpha}u)(\bar{u}\sigma^{o\alpha}u)|_{\text{s,t}}$. Then, T_{qq}^3 and T_{qq}^4 can be related as

$$T_{qq}^3 = -\frac{15}{4}T_{qq}^1 + \frac{9}{4}T_{qq}^2 \tag{C12}$$

$$T_{qq}^4 = -\frac{15}{4}T_{qq}^2 + \frac{9}{4}T_{qq}^1. \tag{C13}$$

T_{qq}^is can be classified as three different classification of K^is in Table III: $K_u^1 = \{K_{ud}^1/\beta,\ K_{ud}^1(\beta+1)/2\beta,\ K_{ud}^1\}$ and $K_u^1 = \{-K_{ud}^1,\ -K_{ud}^1(\beta+1)/2\beta,\ -K_{ud}^1/\beta\}$. T_{qq}^is are classified in Table II according to these three classifications in two sets.

Appendix D: QCD sum rule formulas for $E_{q,V(I)}$ and $E_{V,\rho}^{\text{sym}}$

In this section, we provide the detailed description for

$$E_{q,V(I)} = \frac{\mathcal{N}_{(\rho^0,I^0)}^{n,p} + \mathcal{N}_{(\rho,I^0)}^{n,p}\rho + \left[\mathcal{N}_{(\rho,I)}^{n,p}\rho\right] I}{\mathcal{D}_{(\rho^0,I^0)}^{n,p} + \mathcal{D}_{(\rho,I^0)}^{n,p}\rho + \left[\mathcal{D}_{(\rho,I)}^{n,p}\rho\right] I}, \tag{D1}$$

$$E_{V,\rho}^{\text{sym}} = \frac{1}{4}\rho\cdot\left[\frac{1}{\mathcal{D}_{(\rho^0,I^0)}^p}\cdot\left(-2\mathcal{N}_{(\rho,I)}^p\right) - \frac{\mathcal{N}_{(\rho^0,I^0)}^p}{(\mathcal{D}_{(\rho^0,I^0)}^p)^2}\cdot\left(-2\mathcal{D}_{(\rho,I)}^p\right)\right], \tag{D2}$$

in QCD sum rule formula. In this formula, $\mathcal{N}_{(\rho^m,I^l)}^p$ and $\mathcal{D}_{(\rho^m,I^l)}^p$ are as follows:

$$\mathcal{N}^p_{(\rho^0,I^0)} = -\frac{1}{4\pi^2}(M^2)^2 E_1 \langle[\bar{q}q]_0\rangle_{\rm vac}, \tag{D3}$$

$$\begin{aligned}
\mathcal{N}^p_{(\rho,I^0)} =& -\frac{1}{4\pi^2}(M^2)^2 E_1 \langle[\bar{q}q]_0\rangle_p - \frac{4}{3\pi^2}\vec{q}^2 \langle[\bar{q}\{iD_0 iD_0\}q]_0\rangle_p L^{-\frac{4}{9}} \\
&+ \frac{2}{3\pi^2}(M^2)^2 \langle[q^\dagger q]_0\rangle_p E_1 L^{-\frac{4}{9}} + \frac{4}{\pi^2}\vec{q}^2 \langle[\bar{q}\{\gamma_0 iD_0 iD_0\}q]_0\rangle_p L^{-\frac{4}{9}} - \frac{1}{12\pi^2}M^2 \langle[g_s q^\dagger \sigma\cdot\mathcal{G}q]_0\rangle_p E_0 L^{-\frac{4}{9}} \\
&+ \bar{E}_q \Bigg\{ -\frac{20}{9\pi^2}M^2 \langle[\bar{q}\{\gamma_0 iD_0\}q]_0\rangle_p E_0 L^{-\frac{4}{9}} - \frac{1}{36\pi^2}M^2 \left\langle \frac{\alpha_s}{\pi}[(u\cdot G)^2 + (u\cdot\tilde{G})^2]\right\rangle_p E_0 L^{-\frac{4}{9}} \\
&\qquad + \frac{1}{\pi\alpha_s}\frac{M_N}{2}\left([T^1_{ud} - T^2_{ud}] + [T^1{}_0 - T^2{}_0] - \frac{1}{3}[T^3{}_0 - T^4{}_0]\right) L^{-\frac{4}{9}} \Bigg\},
\end{aligned} \tag{D4}$$

$$\begin{aligned}
\mathcal{N}^p_{(\rho,I)} =& -\frac{1}{4\pi^2}(M^2)^2 E_1 \langle[\bar{q}q]_1\rangle_p - \frac{4}{3\pi^2}\vec{q}^2 \langle[\bar{q}\{iD_0 iD_0\}q]_1\rangle_p L^{-\frac{4}{9}} \\
&- \frac{1}{2\pi^2(M^2)^2}\langle[q^\dagger q]_1\rangle_p E_1 L^{-\frac{4}{9}} - \frac{2}{\pi^2}\vec{q}^2 \langle[\bar{q}\{\gamma_0 iD_0 iD_0\}q]_1\rangle_p L^{-\frac{4}{9}} + \frac{1}{4\pi^2}M^2 \langle[g_s q^\dagger \sigma\cdot\mathcal{G}q]_1\rangle_p E_0 L^{-\frac{4}{9}} \\
&+ \bar{E}_q \left\{ \frac{4}{3\pi^2}M^2 \langle[\bar{q}\{\gamma_0 iD_0\}q]_1\rangle_p E_0 L^{-\frac{4}{9}} + \frac{1}{\pi\alpha_s}\frac{M_N}{2}\left(-[T^1{}_1 - T^2{}_1] + \frac{1}{3}[T^3{}_1 - T^4{}_1]\right)\right\} L^{-\frac{4}{9}},
\end{aligned} \tag{D5}$$

$$\mathcal{D}^p_{(\rho^0,I^0)} = \frac{1}{32\pi^4}(M^2)^3 E_2 L^{-\frac{4}{9}} + \frac{1}{32\pi^2}M^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\rm vac} E_0 L^{-\frac{4}{9}} + \frac{2}{3}\langle[\bar{q}q]_0\rangle^2_{\rm vac} L^{\frac{4}{9}}, \tag{D6}$$

$$\begin{aligned}
\mathcal{D}^p_{(\rho,I^0)} =& -\left(\frac{5}{9\pi^2}M^2 E_0 - \frac{8}{9\pi^2}\vec{q}^2\right)\langle[\bar{q}\{\gamma_0 iD_0\}q]_0\rangle_p L^{-\frac{4}{9}} \\
&+ \frac{1}{32\pi^2}M^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_p E_0 L^{-\frac{4}{9}} + \frac{1}{144\pi^2}\left(M^2 E_0 - 4\vec{q}^2\right)\left\langle\frac{\alpha_s}{\pi}[(u\cdot G)^2 + (u\cdot\tilde{G})^2]\right\rangle_p L^{-\frac{4}{9}} \\
&+ \frac{4}{3}f\cdot\langle\bar{q}q\rangle_{\rm vac}\langle[\bar{q}q]_0\rangle_p L^{\frac{4}{9}} - \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left([T^1_{ud} - T^2_{ud}] - [T^1{}_0 - T^2{}_0] + \frac{1}{3}[T^3{}_0 - T^4{}_0]\right) L^{-\frac{4}{9}} \\
&+ \bar{E}_q \Bigg\{ \frac{1}{3\pi^2}M^2 E_0 L^{-\frac{4}{9}}\langle[q^\dagger q]_0\rangle_p - \frac{4}{3\pi^2}\left(1 - \frac{\vec{q}^2}{M^2}\right)\langle[\bar{q}\{\gamma_0 iD_0 iD_0\}q]_0\rangle_{\rho,I} L^{-\frac{4}{9}} \\
&\qquad - \frac{2}{3\pi^2}\langle[\bar{q}\{\gamma_0 iD_0 iD_0\}q]_0 u\rangle_p L^{-\frac{4}{9}} + \frac{1}{18\pi^2}\langle[g_s q^\dagger \sigma\cdot\mathcal{G}q]_0\rangle_p L^{-\frac{4}{9}} \Bigg\},
\end{aligned} \tag{D7}$$

$$\begin{aligned}
\mathcal{D}^p_{(\rho,I)} =& \ \frac{1}{3\pi^2}M^2 E_0 \langle[\bar{q}\{\gamma_0 iD_0\}q]_1\rangle_{\rho,I} L^{-\frac{4}{9}} \\
&- \frac{4}{3}f\cdot\frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)}\langle\bar{q}q\rangle_{\rm vac}\langle[\bar{q}q]_0\rangle_p L^{\frac{4}{9}} - \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left([T^1{}_1 - T^2{}_1] - \frac{1}{3}[T^3{}_1 - T^4{}_1]\right) L^{-\frac{4}{9}} \\
&+ \bar{E}_q \left\{ -\frac{2}{3\pi^2}\langle[\bar{q}\{\gamma_0 iD_0 iD_0\}q]_1\rangle_p L^{-\frac{4}{9}} + \frac{1}{18\pi^2}\langle[g_s u^\dagger \sigma\cdot\mathcal{G}u]_1\rangle_p L^{-\frac{4}{9}} \right\}.
\end{aligned} \tag{D8}$$

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